Heritage Science

RESEARCH

An investigation on the craft of Tianche: traditional timber salt well structures of the Sichuan Basin, China

Zhenyu Zhu[1,2], Haiqing Li[1,2*] and Jingwei Yu[1,2]

Abstract

This study investigates the craft of *Tianche*, an ancient towering timber structure used in well salt production in Sichuan Province, China. Originating in the Han Dynasty and maturing in the late Qing Dynasty, Tianches can reach heights of up to 110 m and represent a unique craft and structural system that is rare worldwide, yet remains largely unexplored in scholarly research. The Tianches are under threat due to a lack of maintenance and extinction of the craft. This study aims to document and elucidate the procedure and principles of the Tianche craft to help conserve this heritage. The key question is how to employ traditional craft to assemble over 100-m-tall structures from small-sized logs using pre-industrial technology. The study combines anthropological fieldwork and principles of mechanics to analyze the scientific validity and rationality of traditional crafts. The study reveals the principles of the Tianche craft, including mature craft procedure and reliable binding joints. Its advantages, namely self-locking structure, frictional force transmission, and hollow cylindrical columns, enhance the structure's strength and stability and are fundamental for constructing 100-m-tall timber structures. Additionally, the study identifies two limitations of the craft: restricted column diameters and the tendency of the inverted dragon member's tenon to break. This study investigated Tianche timber structures featuring hollow, bundle-like columns with frictional force transmission mechanism, presenting new possibilities beyond mainstream timber structural systems. The findings may contribute to enhancing research on the Tianche craft and could lay the foundation for future conservation practices related to Tianche heritage.

Keywords *Tianche*, Salt well derrick, Timber structure, Chinese fir logs, Craft procedure, Craft principles

Introduction

In China, the Sichuan Basin's salt well production flourished during the Eastern Han Dynasty (25–220). From the early modern period, salt well production in Zigong (自贡, Fig. 1a), a city in Sichuan province, became remarkably prolific. During the Second World War, it supplied one-third of the salt to the Nationalist-controlled areas. This substantial production relied on traditional salt well technology and associated architectural complexes.

Among the structures used for well salt production, the towering salt well derricks, locally termed *Tianche* (Fig. 1b–d), are the most striking. Constructed from thousands of bundled *Cunninghamia lanceolata* logs (commonly known as Chinese fir), these derricks can reach heights exceeding 100 m. They consist of bundled logs forming hollow cylindrical columns that taper upward and interconnect in mid-air [1; p. 316]. These derricks, situated above the wellhead, play a vital role in drilling, brine lifting, and maintenance.

Figure 2 shows the architectural complex of the *Shenhai* Saltworks, with an 18.3-m-tall Tianche (Fig. 2b)

*Correspondence:
Haiqing Li
jianzhu_shi@sina.com
[1] School of Architecture, Southeast University, Nanjing, China
[2] Key Laboratory of Urban and Architectural Heritage Conservation, Southeast University, Ministry of Education, Nanjing, China

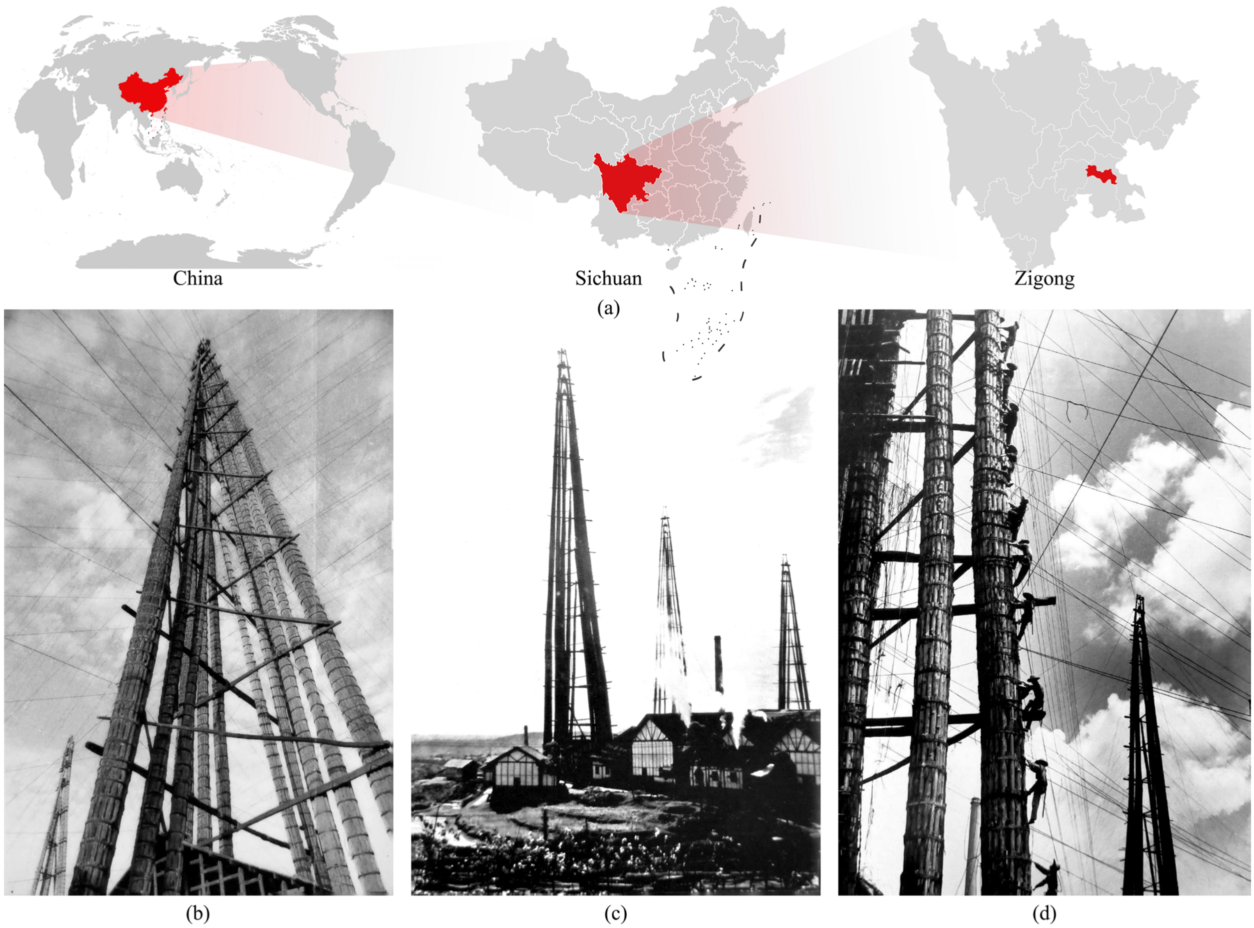

Fig. 1 **a** *Dashisi* Well (大十四井) Tianche (88 m tall); **b** *Dade* well (达德井) Tianche (113.32 m tall); **c** the craftsmen are climbing the Tianche. Sources: **a** the authors' collection; **b**, **c** Mingyuan Yu's photographs taken in the 1950–70 s

standing above the salt well. Oxen at the power workshop (Fig. 2c) harness their strength to draw brine from underground and channel it via bamboo pipelines to the evaporation pans located in the salt-making workshop (Fig. 2d). The heat for evaporation is supplied by natural gas spewing from the well and channeled through bamboo pipelines (Fig. 2f). The production system is constructed entirely using natural materials (for details, see Fig. S1–2 and Additional file 1).

The height of the Tianche exceeds the length of the brine extraction bucket by approximately 10%. The taller the Tianches, the longer the bucket, enabling greater production efficiency. The craft of these towering structures has been refined over centuries through craftsmen's experiences. Instead of using modern design and drawings, craftsmen mentally conceive Tianche's dimensions, decide on the necessary materials, and proceed directly with construction.

Around the year 1916, approximately 10,000 salt wells were estimated to be in operation, under excavated, or abandoned [2]. Each salt well was accompanied by a Tianche. However, after reaching maturity during the Qing Dynasty, Tianche technology plateaued, experiencing localized improvements but overall stagnation. The Sichuan basin's remote geography limited the influx of modern building techniques, allowing the Tianche craft to flourish until the 1960s, essentially becoming an ancient craft relic. It was not until the 1980s that the Tianches were entirely phased out.

Related research primarily covers history and architecture. In the field of history, the focus has been on policies, institutions, taxation, transportation, and contracts related to Sichuan salt wells [3]. Comprehensive studies on ancient drilling and well repair tools, drilling methods, brine extraction, and salt-making techniques have led to significant progress in the understanding of salt well technology [1, 4]. However, these studies predominantly focused on salt-making processes and the wells, overlooking the prominent Tianche structures above the wells.

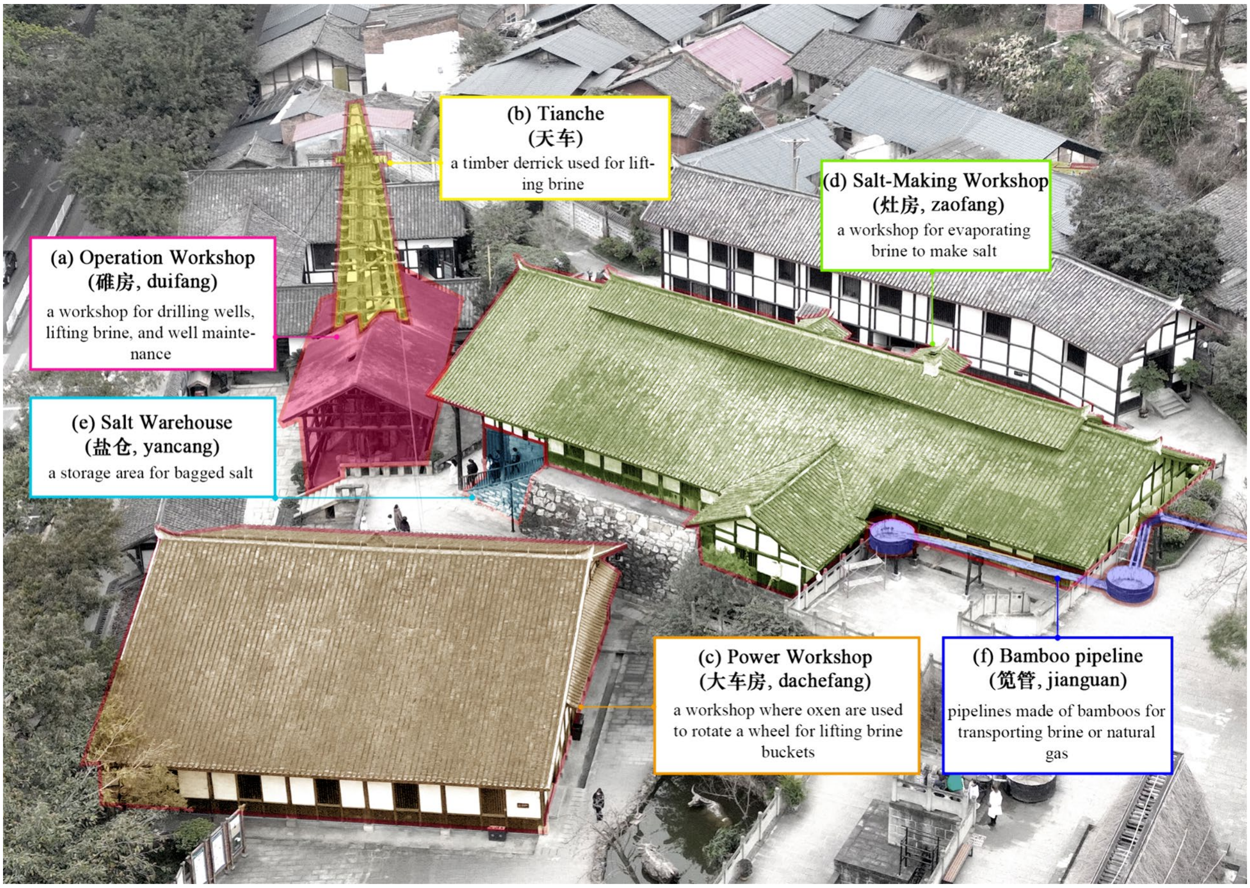

Fig. 2 Shenhai Well (燊海井) architectural complex and a small Tianche, approximately 18.3 m in height. Sources: the authors

In architecture, the Chinese traditional timber structures, which are represented by *large woodworks* (大木作) in the central regions of the Chinese civilization, has been widely studied. Chen comprehensively studied aspects such as *Caifen* (材分), *Puzuo* (铺作), palace hall (殿堂), and mansion hall (厅堂) of the *Yingzao fashi* [5], while Zhang further explored the *standard timber* (基准材) in the *Yingzao fashi* [6]. Additionally, the structural performance [7], the joint characteristics [8], and the seismic behavior [9] of Chinese traditional timber structures have received significant attention. However, the craft of constructing towering structures using small logs has not received significant attention from researchers.

As research has deepened in the central areas of civilization, regional timber structures have garnered increasing attention. Zhang explored the Pile-dwelling timber structures used by Dai (傣), Zhuang (壮), and other ethnic groups [10]. Liu investigated the technological and historical aspects of woven arch bridges (编木拱桥) in the Fujian-Zhejiang region [11]. Chen et al. critically examined the recent construction activities of these bridges, assessing their impact on traditional bridge-building techniques in Fujian and Zhejiang [12] and evaluating the reconstruction of wooden arch bridges as tangible and intangible heritage [13]. Both woven arch bridges and Tianches are large-scale structures composed of logs from the pre-industrial era. While woven arch bridges represent the zenith of traditional timber structures in terms of span, Tianche structures are the pinnacle in terms of height.

The history of the Sichuan salt industry and Chinese traditional timber structures have been extensively studied; however, Tianches have received almost no attention. Urgent research is needed to demystify the procedure and principles of Tianche craft. The pivotal question is how to assemble nonstandard logs to construct timber structures exceeding 100 m in height, and what the essence of this craft entails.

Currently, only 20 Tianches survive in Zigong (Fig. 3), with six designated as part of China's National Key Cultural Relics Protection Units (Table S2. Details on the condition of existing Tianches are given in Additional file 1). As a pivotal element of Sichuan salt well cultural heritage, Tianches remain obscure to non-Chinese

Fig. 3 **a**–**d** Existing Tianches in Zigong are generally 20–40 m tall; **e**–**g** details of the Tianche columns. Sources: the authors

scholars and receive scant attention within China. People remain uninformed about the intricacies of Tanche craft procedure and principles. Consequently, Tianches have been gradually reduced to mere cultural symbols in Zigong's tourism narrative, and the entities responsible for their conservation are losing touch with their craft essence.

The last group of experienced Tianche craftsmen (locally called 辊子匠 or *binding craftsmen*) are now septuagenarians. Knowledge transfer among these craftsmen has traditionally been oral, following a master–apprentice model, without formal documentation. Hindered by a lack of theoretical knowledge, the craftsmen struggle to articulate these building crafts, their understanding being confined to practical experience. This could lead to the extinction of this craft (details on the threats faced by Tianche are given in Additional file 1). The optimal approach to truly understand the craft of Tianche involves apprenticeship and craft analysis.

This study aims to document and elucidate the craft of Tianche, clarifying its procedure and principles to help protect the remaining Tianches and preserve this traditional craft. This research aims to introduce this unique timber structure system to the academic and professional community, potentially enhancing global knowledge, and encouraging further investigation and conservation efforts into this endangered timber structure.

Provenance

Surrounded by mountains, the Sichuan Basin's remote and isolated terrain impedes its connectivity with the outside world. Yet, its fertile interior plains have fostered early civilizations such as *Sanxingdui* (三星堆). This led to the development of advanced building technologies in the basin earlier than nearby regions. The mountains around the basin produce Chinese firs, and the rivers descending from the Tibetan Plateau form efficient transportation channels for these logs. The extensive Tianche construction industry in the region consequently boosted handcrafted bamboo rope workshops [14] (Fig. S4. Details on the sources of materials are presented in Additional file 1). The advanced building techniques, combined with abundant material supply, contributed to the development of the sophisticated timber structural craft of Tianche.

The origin of Tianche can be traced back to salt well scaffolding (Fig. 4a) in the Eastern Han Dynasty (东汉, 25–220). This timber frame structure, designed for large-diameter wells, provides a platform for salt workers. The *Zhuotong Well* (卓筒井), a type of deep well with a diameter of only 5–15 cm and depth reaching more than 1000 m, was invented in Sichuan between the years 1041 and 1053 [15]. Lifting slender brine buckets required taller derricks and likely catalyzed the evolution from salt-well scaffolding to Tianche.

From the mid-eleventh to the early nineteenth century, the Tianche evolved from single- to double- and triple-legged structures (Fig. 4b). The Taiping Rebellion (1851–1864), resulting in a sea salt blockade, spurred significant advancements in Sichuan salt wells. Subsequently, the Tianche structure was rapidly optimized, evolving into the currently known quadruple-legged form and diversifying into six-, eight-, and ten-legged variants (Fig. 4c, d). British explorer William Gill visited Zigong on May 2, 1877, and noticed a 100-feet-tall Tianche [16]. In 1888, American missionary Virgil Chittenden Hart observed 60–160-feet-tall Tianches and also sketched a Tianche structure [17] (Fig. S7). These accounts indicate that Tianche reached nearly 50 m in height in the late nineteenth century. In the early twentieth century, their construction underwent further optimization and expansion of applications, including refining the technique of *doll member* (娃娃料, spindle-shaped elongated rods) (Fig. 4e), widespread adoption of *bending-bar* (扳杆子, two small columns on top of Tianche), and inventing the mechanical-brine-lifting-station (机力升卤站) during the Second World War.

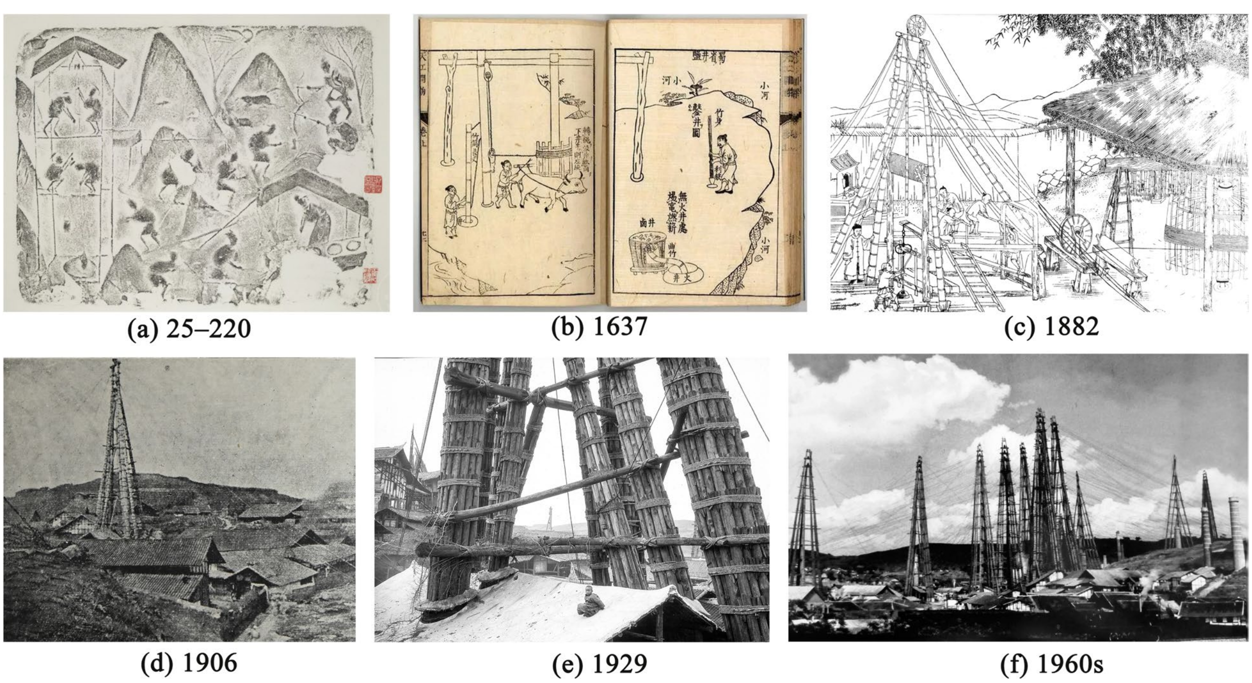

Fig. 4 Structural forms of Tianche during different periods from the Eastern Han Dynasty to the late twentieth century. Sources: **a** Academia Sinica; **b** *Tiangongkaiwu*天工开物; **c** *Sichuanyanfazhi*四川盐法志; **d** *Bashu*巴蜀; **e** ETH archive; **f** Mingyuan Yu

Following the establishment of the People's Republic of China, a public–private partnership mode emerged, which led to the amalgamation of numerous salt merchants into large state-owned salt factories. This era witnessed a rapid increase in the height of Tianche structures (Fig. 4f), significantly boosting production efficiency. The newly constructed Tianche reached staggering heights of 80–90 m [18]. In 1962, the *Dade* Well Tianche was elevated to 113.32 m (Fig. 1c), becoming the tallest in Chinese salt well history [19]. These large Tianches, often expanded from smaller ones, featured 12–14 columns and housed three pulleys for continuous extraction. However, from the 1970s, the development of the *Changshan* salt mine rendered traditional salt well crafts obsolete, gradually phasing out the Tianche.

Having evolved over two millennia, the Tianche is among the tallest timber structures constructed using pre-industrial techniques. It boasts a unique construction craft and structural features, enriching the global landscape of timber structural knowledge and holding significant historical, cultural, and heritage value.

Methods

Methods

This study designed an interdisciplinary research method that integrates anthropology and engineering to understand this traditional craft adequately and incorporate it into the modern knowledge system. First, drawing from anthropological fieldwork approaches, apprenticeship learning, construction experiments, and archival research are used to collect primary data. Next, the procedure and structural concepts within the craft are analyzed to develop an ideal model of Tianche structure. Finally, principles of mechanics are employed to understand the craft of constructing 100-m-tall timber structures (Fig. 5).

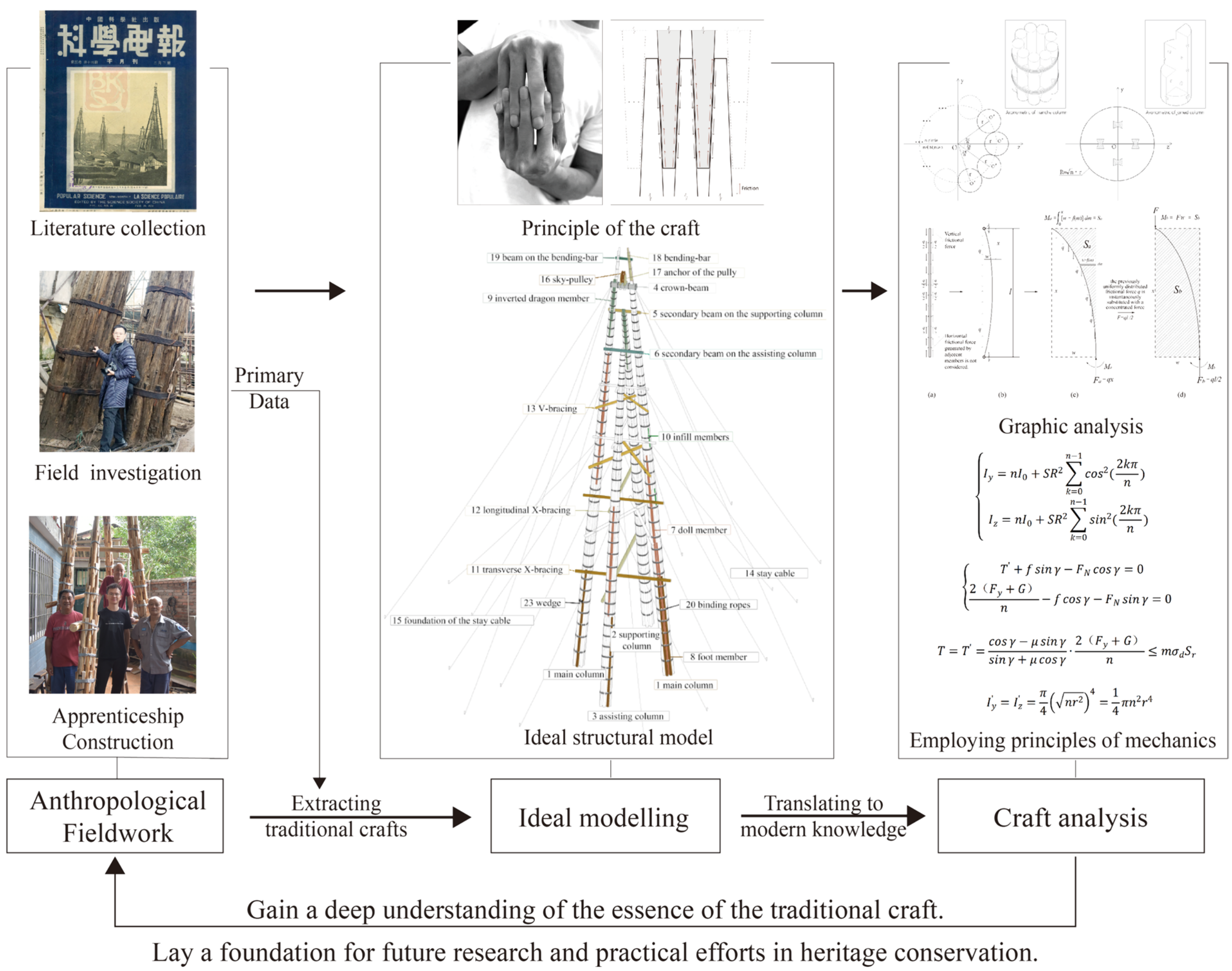

Fig. 5 The research method used in this study. Sources: the authors

Anthropological fieldwork
This study employed anthropological fieldwork, which requires immersing oneself in indigenous communities to learn their languages and cultures. The authors' investigation involved apprenticing and working with Tianche craftsmen, and understanding their dialect and local vocabulary. This method was used to gather primary historical data on the craft through observation, participation, note-taking, and time-lapse photography. This approach is necessary owing to the craftsmen's reliance on oral teaching rather than written documentation, making apprenticeship the only avenue to acquire the intricate details of this craft.

Ideal modeling
Following the acquisition and in-depth understanding of Tianche craft, we extracted craft concepts and established ideal structural models. This approach is necessitated because Tianches were constructed from logs of varying sizes, which presents challenges for structural calculation and analysis.

Craft analysis
This study uses principles of mechanics to analyze the craft, linking traditional craftsmanship with modern knowledge to help heritage conservationists and the public understand this nearly-lost craft.

Experiment
The experimental section of this research was conducted primarily through construction experiments (Figs. 6, 7), drawing on anthropological fieldwork methods.

Experimental materials: 100 small debarked Chinese fir logs, each 1.5–2 m in length and 6–12 cm in diameter; one thick, short timber; white coir ropes; wire ropes with a diameter of 4 mm.

Experimental tools: long-handled axes, Chinese hand saws, Chinese machetes, Chinese bark spuds, Chinese crowbars, hammers, Chinese chisels, ink line reel, waist ropes made of coir ropes, safety ropes made of coir ropes, and a camera for time-lapse photography (Fig. S8).

Construction time: May 23–June 1, 2023.

Construction location: Yuanyue Liang's courtyard in Aiye town (艾叶镇), Gongjing District (贡井区), Zigong City, Sichuan Province.

Participants: master craftsman: Congyou Liu, senior craftsman: Hanzhao Liu, Yuanyue Liang, apprentice: Zhenyu Zhu, author of this paper.

Experiment process: following traditional craft, the structural members were pre-fabricated in 3 days, and the assembly took 7 days. A 5-m-tall Tianche model was constructed (video documentation of the construction procedure are given in Additional file 2). In field testing, this derrick model was capable of lifting four adult persons, collectively weighing 320 kg.

Analysis and results
By combining anthropological fieldwork with principles of mechanics, this study reveals the craft procedure, craft of binding, advantages and limitations of Tianche craft for the first time.

Craft procedure
Tianche construction follows a unique procedure, without scaffolding. The prefabrication of the members precedes their hoisting and assembly. During aerial construction, the binding craftsmen carry only the essential tools: long-handled axe, rattan hat, waist rope, and safety rope (Fig. 8). The craft procedure, subjected to nonstandard logs, has inherent uncertainties yet follows established guidelines.

Prefabricating components
The construction of Tianche requires approximately ten types of timber members, as shown in Table 1 and Fig. 9. Among these, doll member, *foot member* (脚子料, slender members located at the base of the column), and *inverted dragon member* (倒龙料) are particularly crucial (Fig. 10a–c). Doll members are crucial components in Tianche construction and play an important role in transmitting frictional forces. These spindle-shaped rods, characterized by their elongated form with a thick middle section tapering towards the ends, vary in length (from approximately 10 to more than 20 m) and diameter (from approximately 10 to more than 30 cm). The manufacturing procedure involves shaping half-tenons at the broader ends of two logs. Subsequently, these logs are joined, the joint secured with ropes, and the structural integrity enhanced by hammering in two to three wooden dowels (木销钉). This sequence of operations, delineated in Fig. 10d–g, effectively harnesses the natural shape of Chinese fir to assemble larger members.

Column construction
The column construction represents the most challenging stage (Fig. 7). The process is more complicated than simply erecting a circle of foot members followed by several circles of doll members.

After placing the initial foot member, known as the *Qingtianzhu* (擎天柱), the first doll member is installed, followed by the second foot member and the second and third doll members. The primary goal is to ascend to the column's apex swiftly, placing two doll members at the

Fig. 6 Main construction techniques. Sources: the authors

top and installing one inverted dragon member. This strategic approach facilitates early determination of the overall shape. The members are then methodically assembled from the bottom to the top, creating a cylindrical and hollow column. The assembly process hinges on the on-site judgment of the member selection expert (选料师), who determines the sequence and placement of the components, which are then hoisted and assembled by the binding craftsmen.

Techniques such as hammering, chopping, scraping, and sawing (Fig. 6) were employed for fine-tuning the members' positions and the column's shape, gradually achieving a hollow cylinder that narrows toward the top and widens at the base.

Structure construction

The procedure for constructing the overall structure involves distinct phases (Figs. 6 and 9). Focusing on the most common quadruple-legged Tianche as an example, the process commences with erecting two main columns, which are hollow cylindrical structures composed of logs. This is followed by the installation of the crown-beam (天箍头, a sturdy short beam at the top of the structure). Subsequently, the remaining two columns, namely supporting column and assisting column, are constructed.

Fig. 7 Construction of a 5-m quadruple-legged Tianche model. Sources: the authors

The final step involves adjusting the stay cables. More details of the construction procedure are presented in Additional file 1.

Binding craft

Tianche construction does not involve nails; apart from some parts employing mortise-and-tenon joints (Fig. S5. More details are presented in Additional file 1),

Fig. 8 Binding craftsmen and their tools: **a** waist rope; **b** safety rope; **c** long-handled axe. Source: **a** Mingyuan Yu's photograph; **b**–**d** authors' photograph

most of the joints are binding joints. Binding joints can be divided into two types: parallel binding and cross binding (Fig. 11).

Parallel binding

Parallel binding is primarily used to prefabricate doll members and to bind columns (Fig. 11a, b), serving as the principal technique in Tianche construction. In the prefabrication of doll members, two ropes are employed, each wrapped around the members 5–6 times. First, one end of the rope is inserted between the tenons of the two logs. Each loop is secured with the following loop. When applying the final loop, two grooved bamboo wedges (企口楔子) are inserted (Fig. 12a). These grooves guide the rope into the existing loops (Fig. 12b), tightening the rope and securing its end, thereby firmly clamping the start of the rope between the tenons. For the columns, ropes are bound at intervals of about 0.6 m. Each rope is wrapped around the column six times; additional loops may impede the effective insertion of the wedges (Fig. 12c). The binding process begins by creating two initial loops, tucking the start of the rope beneath them, and threading it 2–3 times for tightening. The remaining rope is wrapped successively. Finally, the end of the rope was threaded through the spaces between the rope and column and pulled tightly, with threading and tightening repeated 3–4 times. Finally, wooden wedges (Fig. 11e–h) are inserted, generating substantial tension on the rope and securing the columns tightly.

Cross binding

Cross binding is primarily used for binding the crown-beam, X-bracing (穿), and V-bracing (剪) (Fig. 11c, d). The ropework at the start and end is similar to that in parallel binding. Here, the members form a larger angle, creating larger gaps between the members and ropes; therefore, it is crucial to find tight spots to fix the start and end of the ropes. Finally, wedges are driven at the cross joint of the ropes (Fig. 11c, d).

These methods highlight a key feature of the craft of Tianche binding: the fixing of the start and end of the ropes does not rely on knots but on the frictional force. This is markedly different from binding bamboo structures, which frequently uses knots.

Ideal modeling

Because the Tianche is pulled from top to bottom by multiple stay cables, it is unlikely to completely overturn. The horizontal component of the pulley forces are borne by the main stay cables attached to the crown-beam at the top, allowing the column to be approximated as a purely compressive member. The assembly of Tianche members might seem random at first, but it conceals a set of succinct and effective craft principles.

Collaboration with the craftsmen revealed that they sometimes use the "interlocking" hand gesture to demonstrate the Tianche assembly method (Fig. 13a). By combining hands-on experience with modern structural knowledge, the authors identified three key principles of this craft: self-locking structure, frictional force

Table 1 Translation of traditional terms from local to modern vocabulary. Source: the authors

No	Traditional terminology	Chinese Pinyin	English translation	Modern terminology	Notes
1	正身	Zhengshen	Main column	Load-bearing column	A hollow cylindrical structure, the quadruple-legged Tianche has two, the eight-legged Tianche has quadruple, and so on
2	支杆	Zhigan	Supporting column	Load-bearing column	A hollow cylindrical structure, the quadruple-legged Tianche has one, the eight-legged Tianche has two, and so on
3	倒挂	Daogua	Assisting column	Load-bearing column	A hollow cylindrical structure, the quadruple-legged Tianche has one, the eight-legged Tianche has two, and so on
4	天箍头	Tiangutou	Crown-beam	Main beam	A sturdy short beam at the top of the structure, with a pully on it
5	支杆扒扒	Zhigan papa	Secondary beam on the supporting column	Secondary beam	Secondary beam on the supporting column, circular cross-section
6	倒挂扒扒	Daogua papa	Secondary beam on the assisting column	Secondary beam	Secondary beam on the assisting column, circular cross-section
7	娃娃料	Wawaliao	Doll member	Structural member	Main members of Tianche, spindle-shaped elongated rod, vary in length from approximately ten to over twenty meters and in diameter from about ten to more than twenty centimeters
8	脚子料	Jiaoziliao	Foot member	Structural member	A slender member made of large logs, more than ten meters long, located at the bottom of the column
9	倒龙料	Daolongliao	Inverted dragon member	Structural member	A slender member made of large logs, more than ten meters long, located at the top of the column
10	填箱料	Tianxiangliao	Infill-members	Constructive member	Typically made from offcuts, employed exclusively for constructional purposes—filling gaps within the structure
11	横穿	Hengchuan	Transverse X-bracing	Horizontal bracing	For the reinforcement connecting two main columns
12	顺穿	Shunchuan	Longitudinal X-bracing	Horizontal bracing	For the reinforcement of supporting column and assisting column
13	剪	Jian	V-bracing	Horizontal bracing	For the reinforcement of supporting column and two main columns
14	风篾	Fengmie	Stay cable	Stay cable	A Tianche has many stay cables; one end of each stay cable is tied to the crown-beam, and the other end is anchored to the ground
15	风篾桩	Fengmiezhuang	Foundation of the stay cable	Foundation of the stay cable	Used to anchor the stay cables
16	天辊子	Tiangunzi	Sky-pulley	Pully	Used for hoisting the brine buckets
17	天夹板	Tianjiaban	Anchor of the pully	Anchor	Used to secure the pulleys to the crown-beam
18	扳杆子	Banganzi	Bending-bar	Bar	Reinforcement component at the top of the Tianche, also serving as a small crane
19	囫圇棒	Huluebang	Beam on the bending-bar	Auxiliary beam	Reinforcement component at the top of the Tianche, also serving as a small crane
20	箍箍	Gugu	Binding rope	Binding rope	A single rope is referred to as a '箍箍'
21	篾索	Miesuo	Bamboo rope	BAMBOO rope	Before the 1960s, Tianches commonly employed bamboo ropes for bundling
22	钢丝绳	Gangsisheng	Wire rope	Wire rope	After the 1960s, Tianches predominantly used steel ropes for bundling
23	楔子	Xiezi	Wedge	Wedge	Driven into the ropes to generate tension

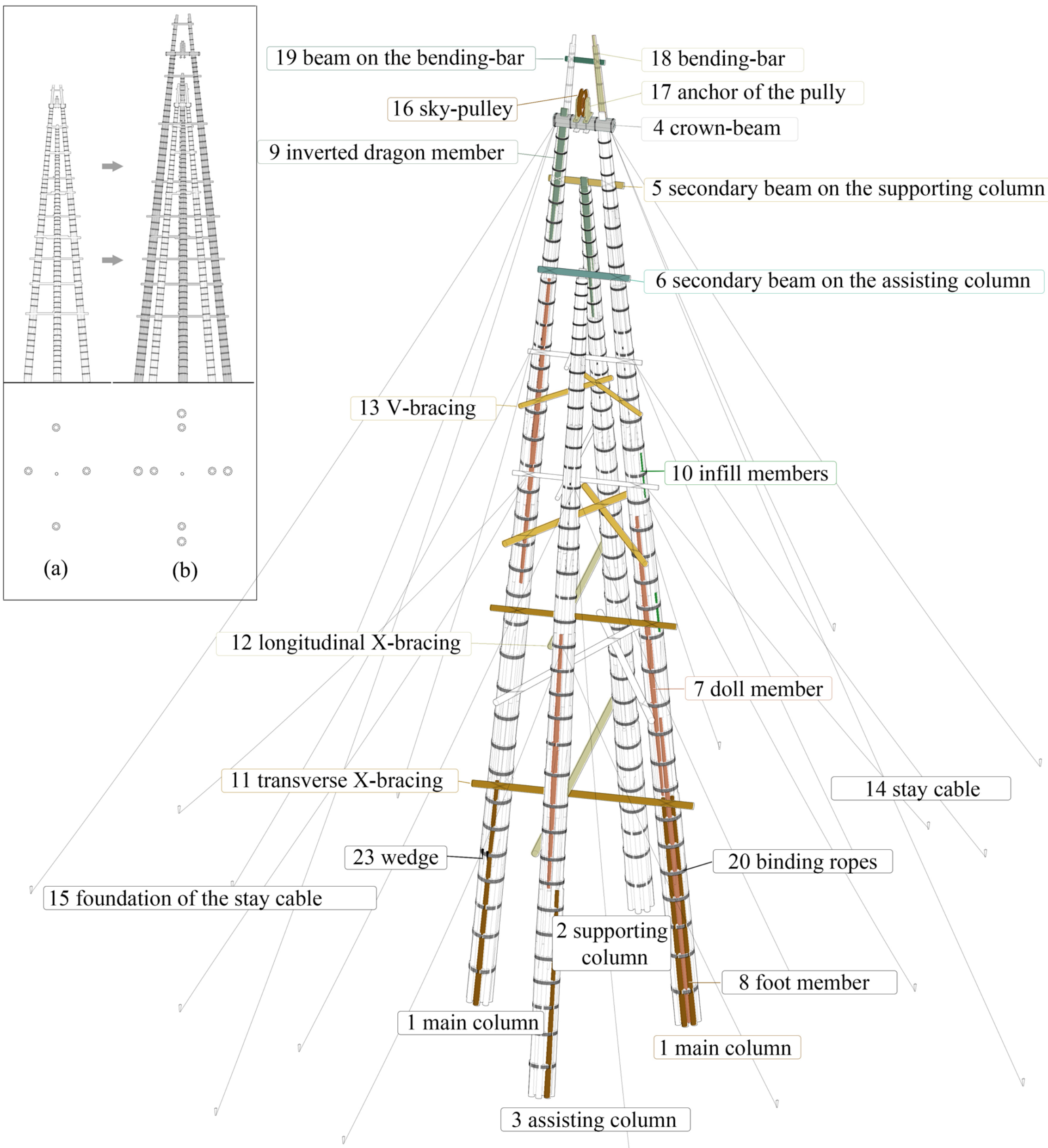

Fig. 9 **a**, **b** Larger Tianche can be expanded from a smaller one; **c** structure and member names of a quadruple-legged Tianche, with the numbers in the figure corresponding to the listing in Table 1. Sources: the authors

transmission, and hollow cylindrical column. This leads to an ideal column-structure model (Fig. 13e–h). In this model, the dimensions of all the members are uniform. The assembly process is as follows.

1. Form a circle with $n/2$ foot members and place them on the ground (Fig. 13e).
2. Form a circle with $n/2$ doll members and insert them into the foot members (Fig. 13f).
3. Successively interlock several circles of doll members, totaling m circles (Fig. 13g).
4. Replace x doll members in the top circle with a single inverted dragon member (Fig. 13h).

The aforementioned assembly process delivers a system of members with inclined contact surfaces that rely on friction to transmit loads. This ideal structural model

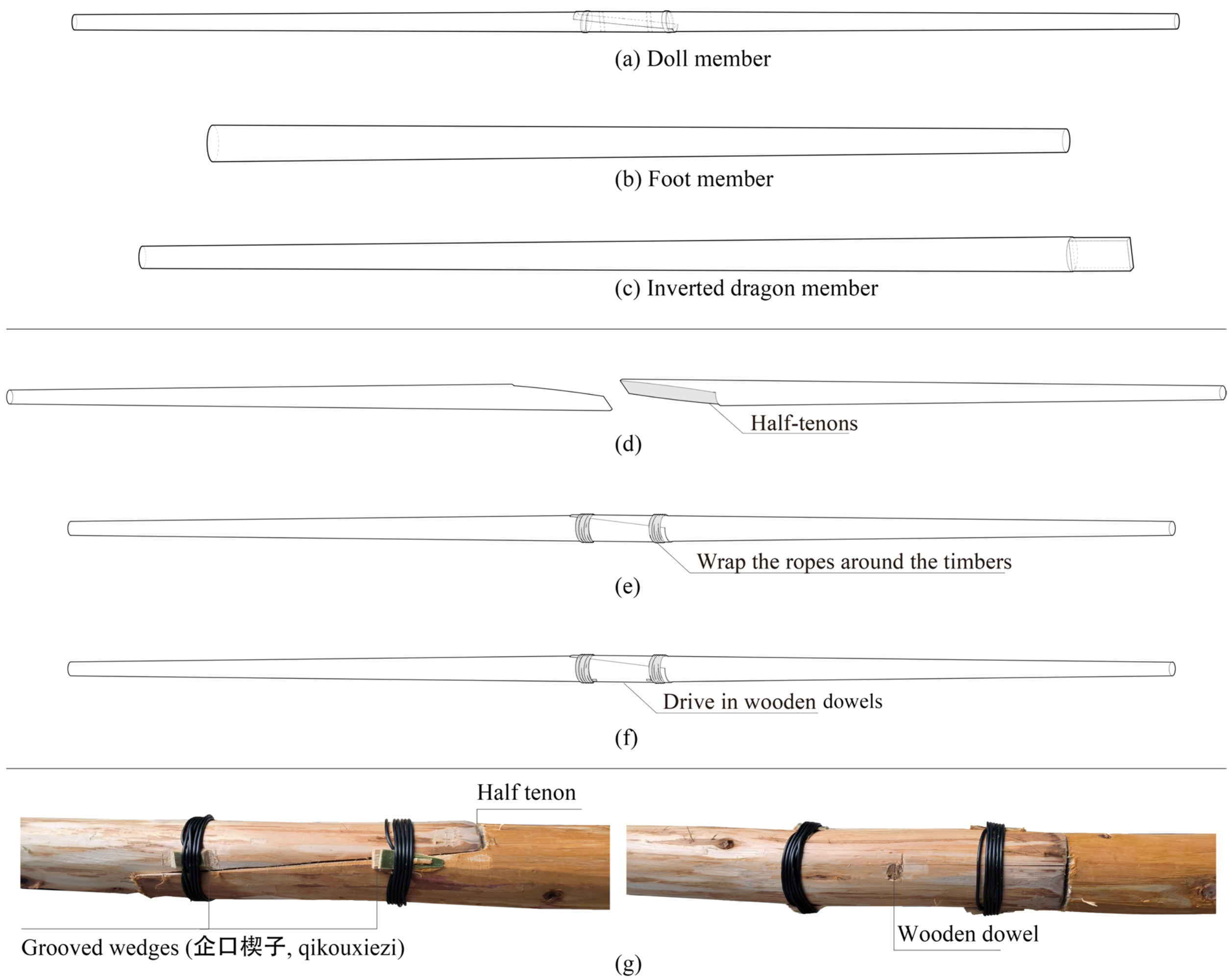

Fig. 10 **a**–**c** Three main members of Tianche; **d**–**f** method of prefabricating doll members; **g** mortise-and-tenon joint and binding joint of a doll member. Sources: the authors

serves as the foundation for analyzing the principles of the craft.

Advantages of Tianche craft

Self-locking structure

Tianche is a type of structure that can lock its displacement and maintain stability when subjected to external loads. The combination logic of the column members is interlocking wedging, where adjacent members become wedges to each other. With repeated dynamic loading, once a member is displaced downward, it acts like a wedge being inserted, increasing the tension in the ropes and consequently increasing the rigidity of the structure (Fig. 13c). Thus, the compressive force from adjacent members also increases, preventing further downward displacement.

It is necessary to maintain a distance of approximately 100 mm between the ends of the upper and lower members to prevent direct contact (Fig. 13d); similarly, a gap should be left between the ends and ground (Fig. 13d). The craft ensures space for the members to displace downward, facilitating the self-locking mechanism. Its significance lies in the ability to counteract the inherent disassembly tendency of large timber assemblies.

Frictional force transmission

The slenderness ratio of the doll members is approximately 80:1 to 100:1, making them prone to buckling under concentrated loads. However, craftsmen use inclined surfaces to increase the contact area between members, dispersing the load into frictional forces to enhance the stability of the members. This section

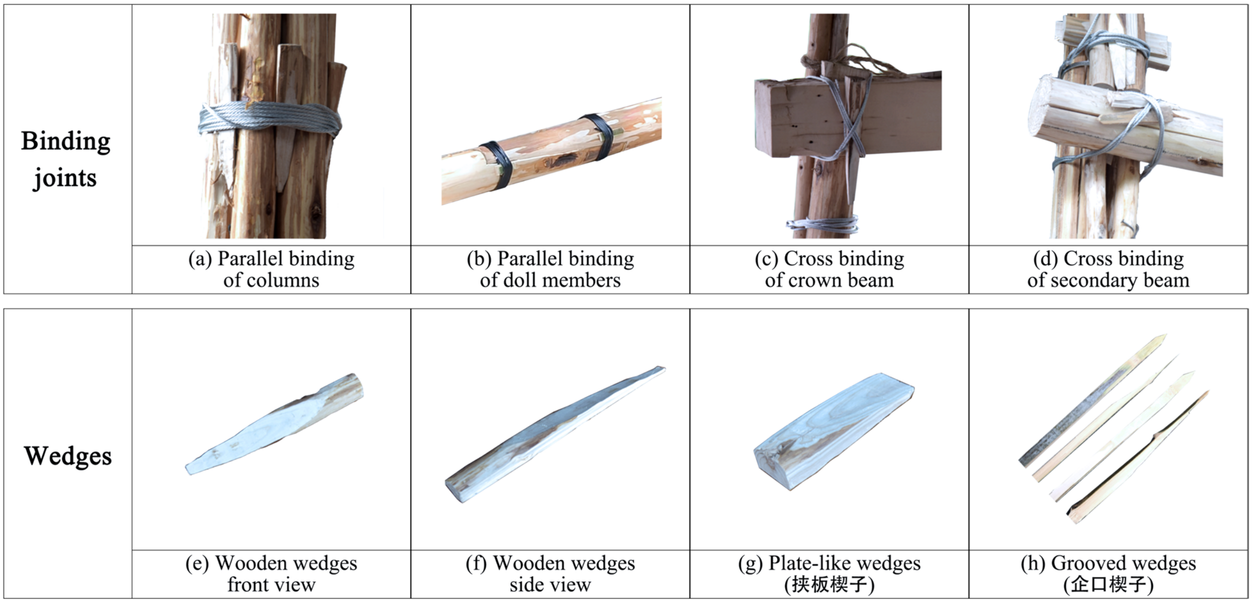

Fig. 11 Binding joints and wedges used in constructing a Tianche. Sources: the authors

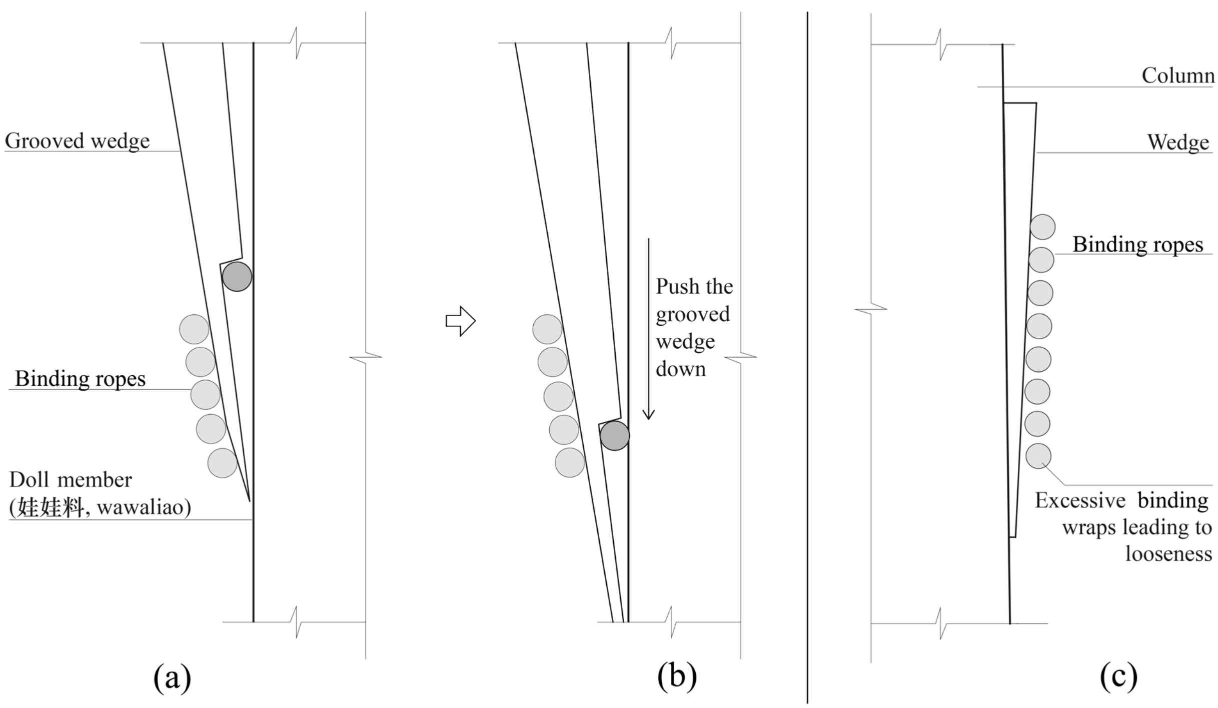

Fig. 12 **a** Excessive binding wraps leading to looseness; **b**, **c** grooved wedges allow the final rope to be brought under other ropes. Sources: the authors

investigates the effect of dispersing concentrated forces at the ends of rods into uniform forces on the stability of the members. This study assumes an ideal mode of loading without clamps (Fig. 14a). The horizontal frictional forces generated by adjacent components and constraints produced by clamps on the member are not considered (Fig. 14a). This allows for a precise analysis of the member's stability when solely subjected

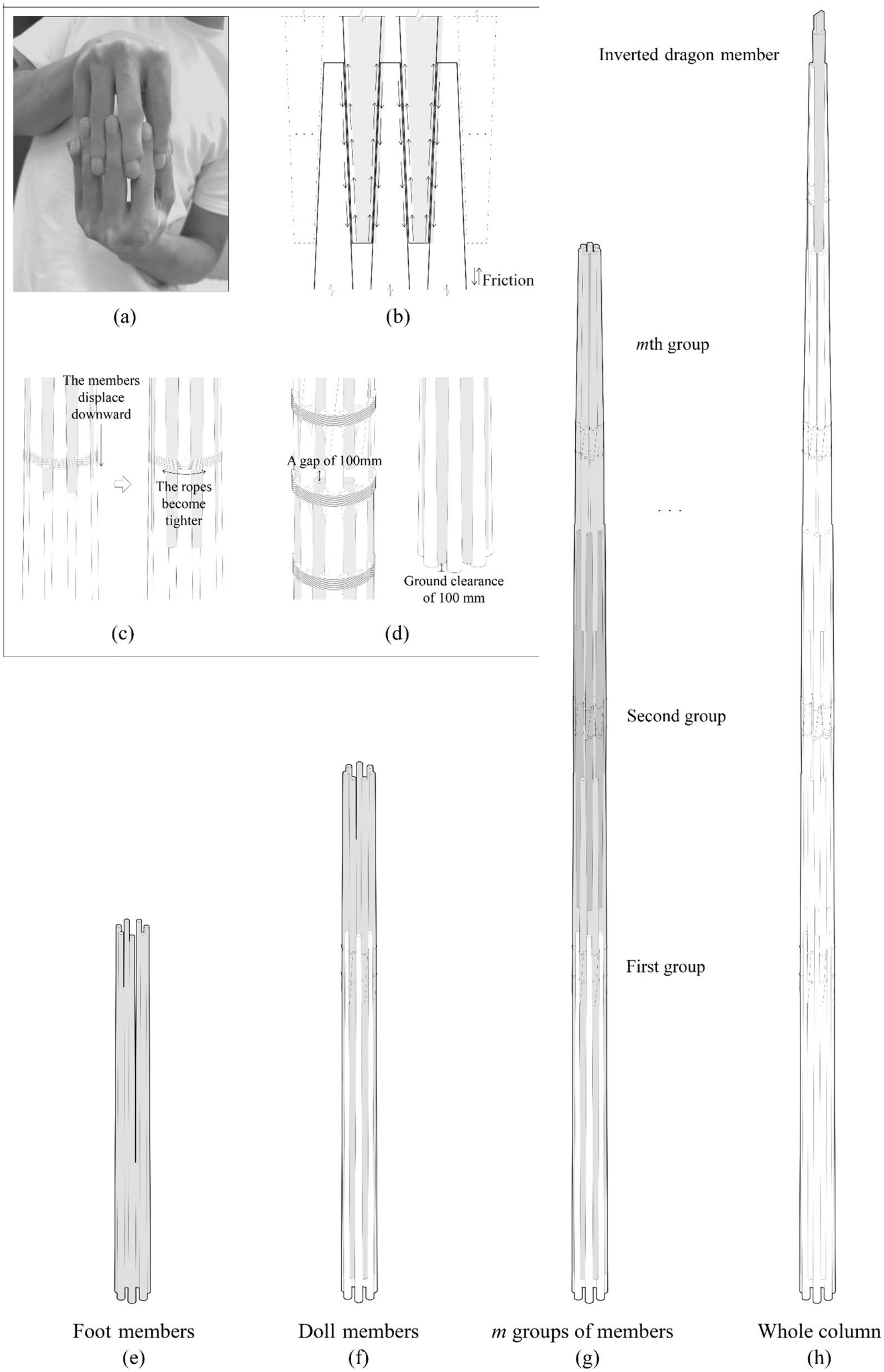

Fig. 13 **a** Hand gesture representing the combination principle of Tianche members; **b** principle of frictional force transmission in Tianche members; **c** members displaced downward tightening the ropes; **d** leaving a gap at the end of the members; **e**–**h** ideal structural model of the column. Sources: the authors

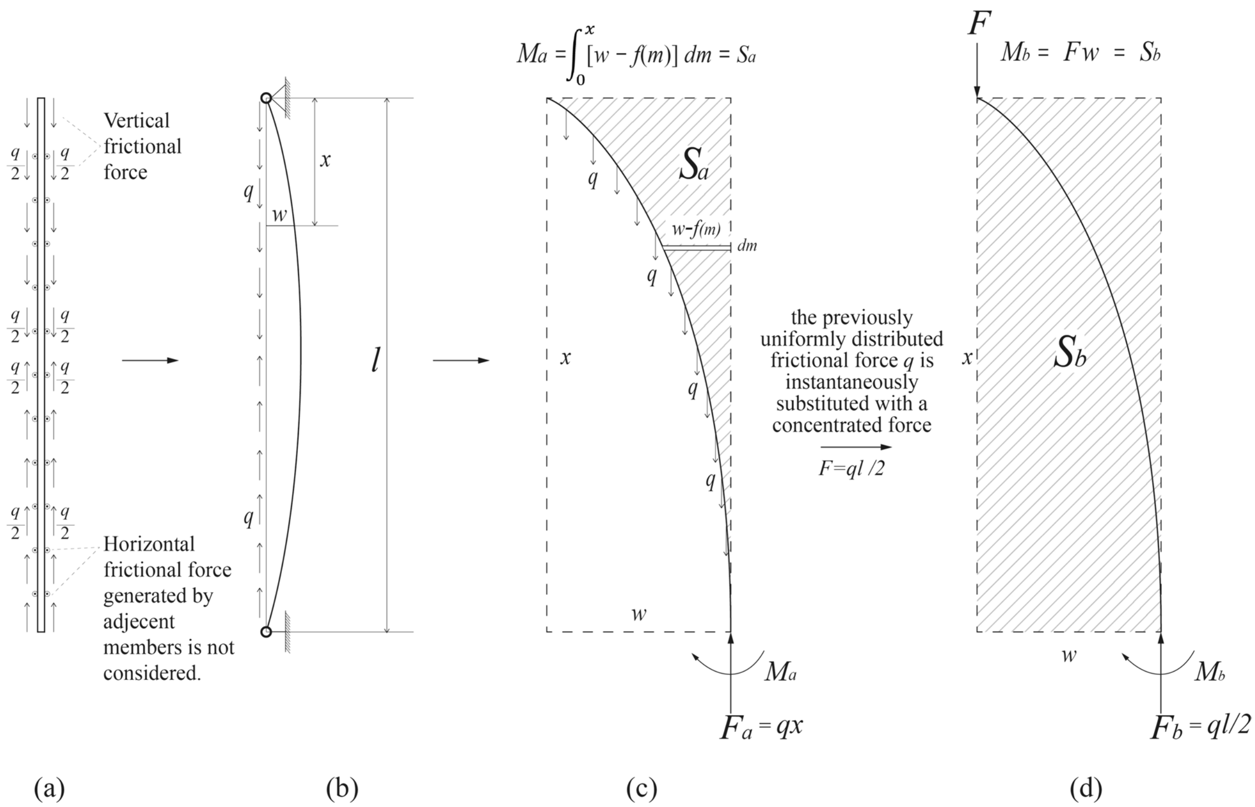

Fig. 14 A compression member under uniform friction (**a**–**c**) or concentrated force (**d**) at the end of a rod. Sources: the authors

to vertical uniform forces. Because of the difficulty in obtaining an analytical solution for the differential equation of the deflection curve when the member is subjected to a uniform force (Fig. 14b, c, upper part), graphical analysis was used (Fig. 14c, d).

Assume a slender rod L with a constant cross-section and length l, and the two ends of the rod are pin-supported. As shown in Fig. 14a, a uniform force $q/2$ (N/m) is applied on both sides along the rod, resulting in a total load of $ql/2$. The force model and deflection curve of rod L are shown in Fig. 14b. Assume that this uniform force brings rod L to a critical state of buckling. We analyze a section of the rod with length x $(0 < x \leq 0.5l)$, as shown in Fig. 14c. The graphical representation of the bending moment M_a is given by the shaded region with area S_a.

As shown in Fig. 14d, the uniform force is instantaneously substituted with a concentrated force $F = ql/2$. The graphical representation of the bending moment M_b is given by the shaded rectangle with area S_b.

It is evident that $S_b > 2S_a$, and thus $M_b > 2M_a$. The bending moment sharply increases by more than two times, indicating that the critical compression member L rapidly descends into buckling. This indicates that the mechanism of transferring loads through friction in a Tianche contributes to the stabilizing effect. This study did not consider lateral frictional resistance generated by adjacent components, which create even more unfavourable conditions; thus, the conclusions drawn here are even more reliable.

Tianche materials must be selected such that the fir logs are thick at one end and thin at the other (Fig. 10). The tenons at the joints of the doll members must be cut at an angle. These conditions ensure that the Tianche transforms concentrated loads into frictional force.

Hollow cylindrical column

Tianches are constructed by assembling smaller logs into larger columns, a method that bears resemblance to the *joined column* (合柱) craft recorded in the Song Dynasty engineering manual, *Yingzao Fashi* [20] (Fig. 15). The dovetail tenon technique of joining columns, also widely used in wooden well casing in Zigong city, has not been used in Tianche construction. This may be attributed to the larger sectional moment of inertia of Tianche compared to the joined columns. The following calculation reveals the differences between these two construction crafts.

Figures 15a, b shows the cross-section of a Tianche column, composed of n $(n \geq 3,\ n \in \mathrm{N})$ circles with radius r, evenly distributed around a large circle. A coordinate system was established with the center of the large circle as

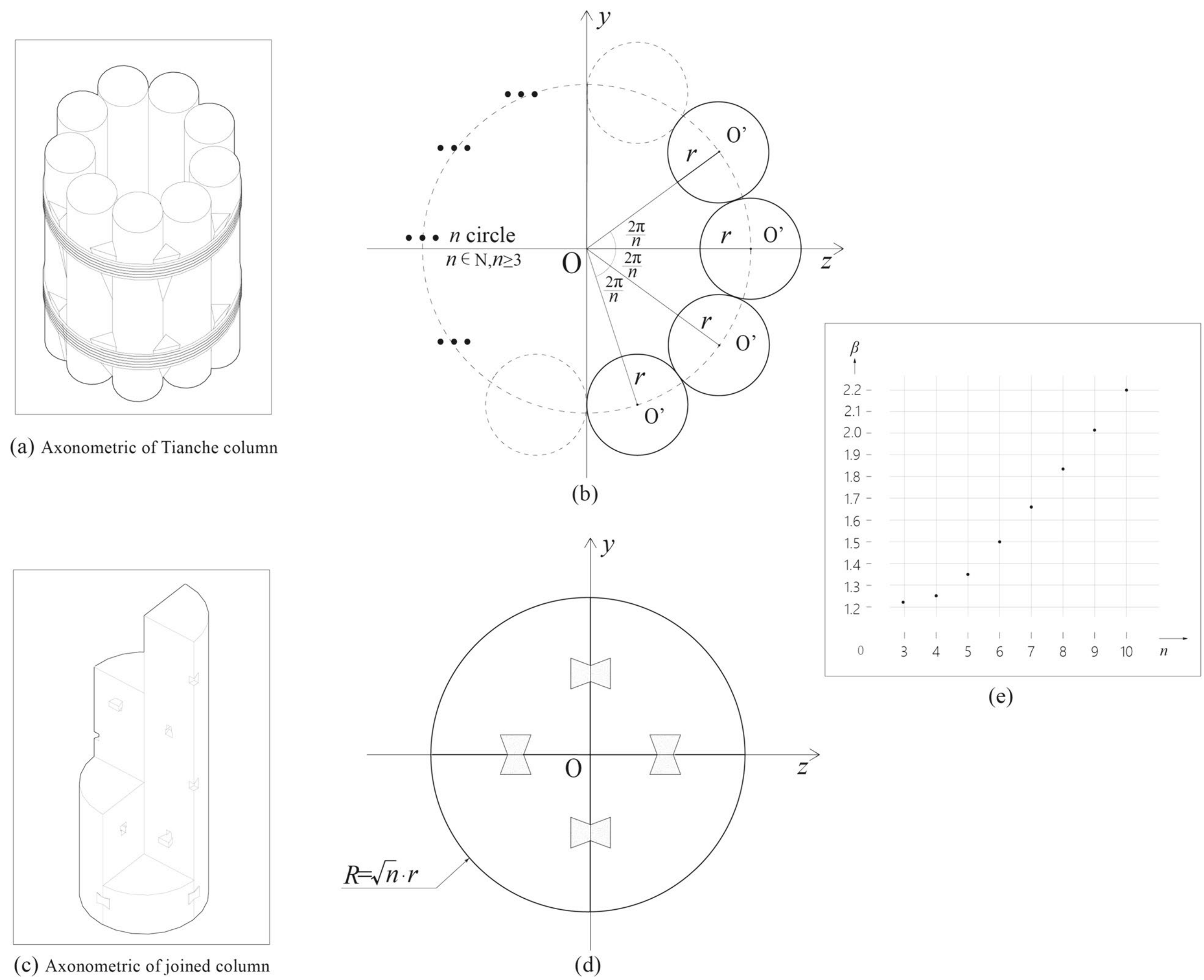

Fig. 15 Cross-sections of the Tianche column and the joined column in *Yingzao fashi*. Sources: the authors

the origin. Figures 15c, d shows the joined column cross-section, with an area equal to that in Fig. 15b. According to the parallel axis theorem:

$$\begin{cases} I_y = nI_0 + SR^2 \sum_{k=0}^{n-1} cos^2\left(\frac{2k\pi}{n}\right) \\ I_z = nI_0 + SR^2 \sum_{k=0}^{n-1} sin^2\left(\frac{2k\pi}{n}\right) \end{cases}, \quad (1)$$

where $R = \frac{r}{sin\frac{\pi}{n}};\ S = \pi r^2;\ n \in N,\ n \geq 3.$

The sectional moment of inertia of the Tianche column is

$$I_y = I_z = n\pi r^4\left(\frac{1}{4} + \frac{1}{2sin^2\frac{\pi}{n}}\right). \quad (2)$$

The sectional moment of inertia of the joined column is

$$I'_y = I'_z = \frac{\pi}{4}\left(\sqrt{nr^2}\right)^4 = \frac{1}{4}\pi n^2 r^4. \quad (3)$$

Therefore, the ratio of their sectional moments of inertia is

$$\beta = \frac{I_y}{I'_y} = \frac{I_z}{I_{z'}} = \frac{I_p}{I'_p} = \frac{1 + \frac{2}{sin^2\frac{\pi}{n}}}{n}. \quad (4)$$

β is an increasing function of the number of members n ($n \geq 3$, $n \in$ N) (Fig. 15e). The higher the number of members in the column, the more apparent is the comparative advantage of the Tianche craft over joined columns in terms of sectional moments of inertia and bending stiffness.

This explains why a small Tianche can be built using a column made of a single log, whereas a large Tianche is built following a hollow approach. It also explains the proverb passed down through generations among the binding craftsmen: "No hollowness, no roundness; no roundness, no bearing strength (不空不成圆，不圆不承力)." This proverb summarizes the relation between structural form and bearing strength: hollowness–roundness–bearing strength.

Based on the aforementioned calculation, hollowness is an important prerequisite for bearing strength. When constructing the Tianche, the authors employed techniques such as hammering and prying to ensure column roundness. The components continuously contract inward owing to the continuous wedging and tightening of the binding. If solid columns are used, there will be no space for inward contraction, and adjustment techniques cannot be applied. Therefore, hollowness is also an important prerequisite for roundness. The aforementioned proverb, although derived from craftsmen's experience, is in agreement with modern structural principles.

Limitations of Tianche craft

Fracture of tenons and unearthing a forgotten craft

The *Dongyuan* Well Tianche is distinct from all other existing Tianches in that two additional timbers are tied at the top (Figs. 3a and 16c). Historically, there have been multiple incidents where the inverted dragon's tenon broke, and the crown-beam and sky-pulley fell from a high height. Such incidents occurred only during the retrieval of objects from a well, not during brine extraction, because of the greater tensile force generated during retrieval. According to the recollections of these veteran craftsmen, the Dongyuan Well has experienced several such incidents. Each time the Tianche was repaired, the craft of *Teng Daolong* (腾倒龙) was employed, which required craftsmen to climb the Tianche and create a new tenon for the inverted dragon member. The two timbers at the top are called *inverted bending-bars* (倒扳杆子) (Fig. 16c). They are temporarily installed before retrieving objects from the well and reinforce the inverted dragon member's tenon. The following calculation explains this forgotten craft.

The bending moment of a single inverted dragon member's tenon is given by

$$M = 2F_t cos\frac{\alpha}{2} sin\frac{\alpha}{2} \cdot D/2. \tag{5}$$

The maximum stress at the bottom of the inverted dragon member's tenon is given by

$$\sigma_{max} = \frac{My_{max}}{I} \leq \sigma_s. \tag{6}$$

Rope tension is

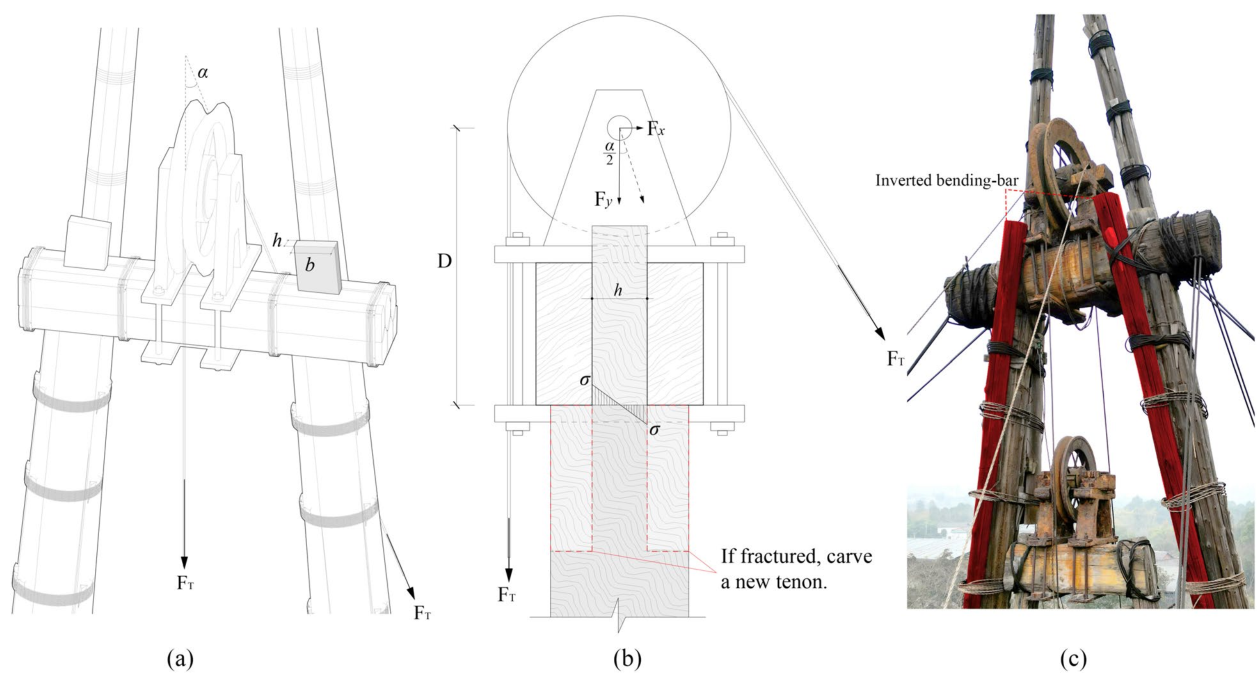

Fig. 16 **a** The angle of the ropes on both sides of the sky-pulley is approximately 30–50°; **b** side view of the shy-pulley, crown-beam, and inverted dragon tenon; **c** top view of the Dongyuan Well Tianche. Sources: the authors

$$F_t \le \frac{bh^2\sigma_s}{6D\,cos\frac{\alpha}{2}sin\frac{\alpha}{2}}, \tag{7}$$

where $30 \le \sigma_s \le 50$ MPa [21; p. 404].

The approximate values of the parameters are: $D=0.8$–1.2 m, $b=0.4$ m, $h=0.2$ m, and $\alpha=30$–$50°$. Assuming $D=1$ m, $b=0.4$ m, $h=0.2$ m, $\alpha=45°$, and $\sigma_s=30$ MPa, gives $F_t \le 226.3$ kN, which is equivalent to a mass of 23.1 metric tons. This aligns with the craftsmen's experience of a lifting limit of 20–30 metric tons [22]. This result can be explained as follows:

1. During brine extraction, the tensile force generally does not exceed 10 metric tons, which is insufficient to damage the inverted dragon member's tenon.
2. When retrieving objects from a well, the tensile force can reach tens of metric tons, potentially damaging the inverted dragon member's tenon. Therefore, the inverted bending-bar must be bound and stay cables added on the sky-pulley for reinforcement.

Ropes versus timber and phasing out of bamboo

Based on the experiences of craftsmen, bamboo ropes tend to age after a few years, while Chinese fir remains robust for decades. However, the craftsmen could not conclude whether timber or ropes would fail first when a Tianche is subjected to a large load. This study used the aforementioned ideal structural model to investigate this issue.

Figure 9c shows a four-legged Tianche in which the main columns solely support vertical forces whereas the horizontal forces are borne by the stay cables. The column height is H, has uniform thickness, and subjected to a vertical force F_y at the top. Each main column is made of $n/2$ pieces of foot members, and each layer of doll members also consists of $n/2$ pieces, resulting in n pieces in the cross-section (Figs. 13e–h and 15b). Each doll member has an average radius of r, the density of Chinese fir is ρ, and the surface friction coefficient is μ. The self-weight of auxiliary components and the tilt angle of the columns is not considered.

The cross-sectional area of the column is $S = n\pi r^2$, and the self-weight at a height of x is $G = \rho S(H - x)g$.

The maximum normal stress in the column section is

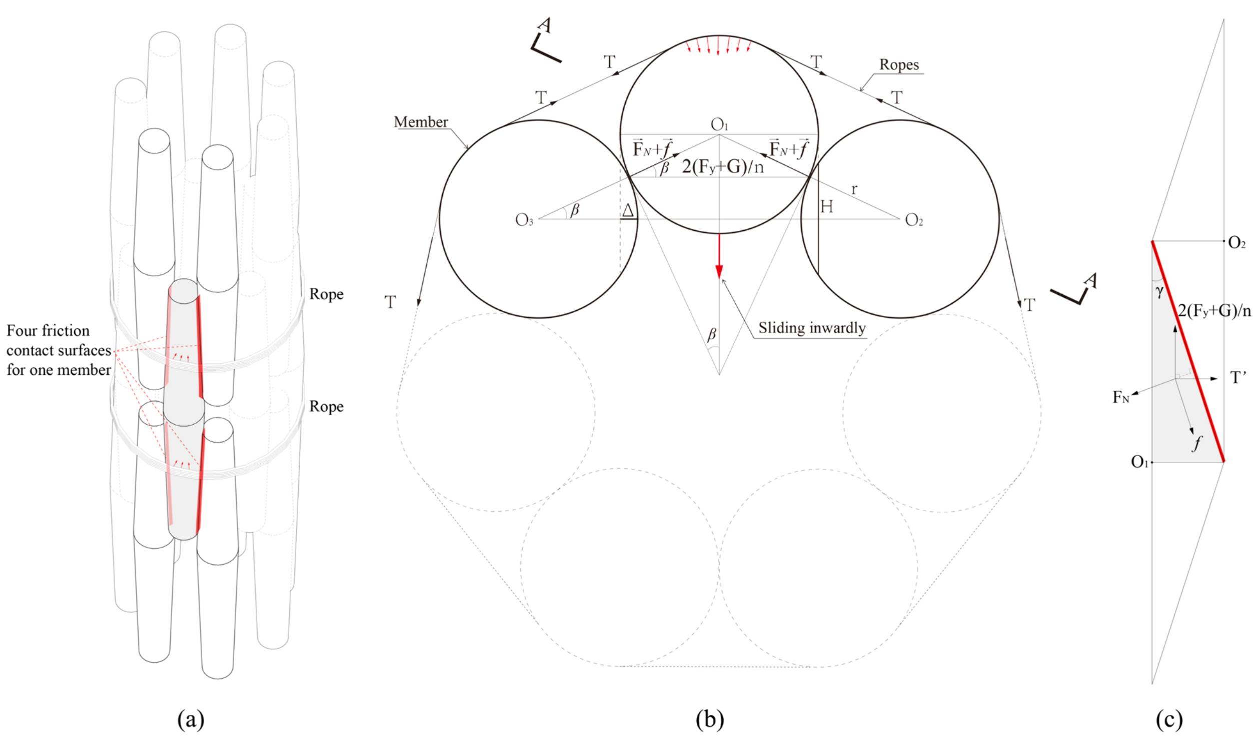

Fig. 17 **a** Stress state of a single doll member; **b** geometry and relationships between adjacent members; **c** simplified stress model of section A–A. Sources: the authors

$$\sigma_{max} = \frac{F_y}{n\pi r^2} + \rho Hg \leq \sigma_s, \tag{8}$$

where $30 \leq \sigma_s \leq 50\,\text{MPa}$ [21; p. 404].

The vertical force at the top of the column is

$$F_y \leq (\sigma_s - \rho Hg)n\pi r^2. \tag{9}$$

The density of Chinese fir is 480–560 kg/m^3 [21; p. 403]. Assuming $\rho = 500$ kg/m^3, $n = 14$, $H = 80$ m, $r = 0.1125$ m (for the doll members, if the diameter at the thickest part is 0.3 m, at the thinnest part 0.15 m, and the average diameter 0.225 m), and $\sigma_s = 30\,\text{MPa}$ gives, $F_y \leq 16,481\,\text{kN}$ and $F_t = \frac{F_y}{(cos22.5°)^2} \leq 19,309\,\text{kN}$; the latter is equivalent to a mass of 1968 metric tons.

Analyzing the upper half of a single doll member (gray area in Fig. 17c) gives

$$\begin{cases} T' + f\ sin\gamma - F_N\ cos\gamma = 0 \\ \dfrac{2(F_y + G)}{n} - f\ cos\gamma - F_N\ sin\gamma = 0 \end{cases}. \tag{10}$$

The tension of the binding rope is

$$T = T' = \frac{cos\gamma - \mu\ sin\gamma}{sin\gamma + \mu\ cos\gamma} \cdot \frac{2(F_y + G)}{n} \leq m\sigma_d S_r, \tag{11}$$

where γ is the slope angle at the thick and thin ends of the Chinese fir, $\frac{2(F_y+G)}{n}$ is the load transferred by a single doll member, and S_r is the cross-sectional area of a single rope.

As γ is very small, T can be approximated as:

$$T = \frac{2(F_y + G)}{\mu n} \leq m\sigma_d S_r. \tag{12}$$

The vertical force at the top of the column is given by

$$F_y \leq \frac{\mu n m \sigma_d S_r}{2} - \rho H n \pi r^2 g. \tag{13}$$

According to craft requirements, we set $n = 14$, $\mu = 0.5$ [23], $\rho = 500$ kg/m^3, $H = 80$ m, $r = 0.1125$ m, and $m = 120$ (assuming that half of each doll member is approximately 12 m long, and bound with 6 loops at intervals of 0.6 m; assume that the strength of timber is infinite).

Since the 1950s, Tianche have been constructed using 7×7 steel wire ropes with a linear density of 1 kg/m^3 and breaking strength of approximately 163 kN [24]. Accordingly, $F_y \leq$ 68,242 kN and $F_t = \frac{F_y}{(\cos 22.5°)^2} \leq 79,950\,\text{kN}$, the latter being equivalent to a mass of 8150 metric tons. Before the 1950s, Tianche craft utilized bamboo ropes with a maximum diameter of 26 mm[25]. The ropes were composed of three strands of solid bamboo ropes with a diameter of approximately 13 mm. Because craftsmen do not make these bamboo ropes anymore, no tensile strength tests could be performed. Therefore, this study employs the tensile strength of bamboo fibers, which is 503 MPa [26]. This gives $F_y \leq$ 83,905 kN and $F_t \leq 98,300\,\text{kN}$, which is equivalent to a mass of 10,020 metric tons.

Therefore, the breaking force of thick bamboo ropes is not necessarily inferior to that of steel wire ropes. The replacement of bamboo ropes with steel wire ropes in the 1950s could have been due to the poor durability of bamboo materials or because the modulus of elasticity of bamboo is smaller than that of steel. Under various weather conditions, timber absorbs water and expands or dries and shrinks. This can loosen the structure; in such cases, steel wire ropes might be more reliable. Details on the durability of Tianche structures are given in Additional file 1.

Inward sliding of members and limited column diameter

The limitation on the column diameter arises due to the increasing number of members, which widens the angle formed by the centers of the three adjacent members. This widening weakens the constraints provided by adjacent members, potentially causing inward displacement of the members. This constraint can be represented by the thickness of the material outside the shear plane between adjacent components, denoted as Δ (Fig. 17b):

$$\Delta = r - (2r\ cos\beta - r) = 2r\left(1 - cos\frac{\pi}{n}\right), \tag{14}$$

where $n \in N, n \geq 3$.

When $n = 18$, $\Delta = 0.03$r; and when $n = 30$, $\Delta = 0.01$r. Even with large doll members with a 0.15 m radius, Δ is only 1.5 mm, leading to the risk of members sliding inward.

Discussion and conclusion

Discussion

From the perspective of craft procedure, binding craft, and craft advantages, the Tianche timber structure, characterized by self-locking, frictional force transmission, and hollow cylindrical column, is unique to Sichuan. Previous research on traditional Chinese timber structure reveals that the finely processed large woodwork of the north achieved significant accomplishments. The woven arch bridges [27], built with logs, represent the pinnacle of what ancient timber structures could achieve in terms of span. Likewise, the Tianche symbolizes the zenith in terms of height. Both are milestones in the history of engineering. Structural sketches of woven arch bridges can be found in Leonardo da Vinci's *Codex Atlanticus* [28]. However, structures similar to Tianche are virtually absent globally, and seldom found in civil engineering publications.

The analysis of craft limitations revealed that, compared with steel wire ropes, the Chinese fir would fail first. Based on the parameter values used in this study, the inverted dragon member's tenon of the Tianche can support approximately 23 metric tons of load, whereas the Chinese fir can support 1968 metric tons and steel wire ropes can support 8150 metric tons. Therefore, the tenon will undoubtedly fail first and unload instantly, ensuring the safety of the main structure. Thus, the Tianche structure maintains an adequate safety margin. Lastly, the inverted dragon member's tenon was found to be the weakest joint, and it determines the maximum load-bearing capacity of a Tianche. Based on the documentation and analysis, this study may contribute to conservation research and practice in this field.

Investigating traditional crafts and architectural heritage in remote areas can be challenging because of the scarcity of textual documentation. This study combines anthropological and engineering research methods, offering a potentially referable approach for similar investigations.

This study presents a detailed investigation of the Tianche craft, a unique timber structure system, which may advance research on hollow, bundle-like columnar timber structure based on the mechanism of frictional force transmission. The findings of this study could enhance understanding of current mainstream timber structure systems. This study introduces a novel comparison of two traditional Chinese column-joining crafts, illustrating the differences between them with formulae. Recording and analyzing the Tianche craft may support the conservation of its heritage (several considerations for conservation are discussed in Additional file 1).

This study has provided a theoretical interpretation of the Tianche craft. Due to uncertainties in the dimensions and shapes of Tianche components, developing an ideal structural model and elucidating its principles may be more crucial than load testing. However, this study has not yet fully documented and explained the Tianche craft. In the future, we aim to advance this study from two perspectives: (1) conducting more interviews and carrying out more construction experiments to explore forgotten crafts; (2) implementing load testing and numerical simulations.

Conclusion

This study followed an apprenticeship approach to document the Tianche craft and analyze its advantages and limitations using principles of mechanics. The craft uses binding joints to assemble logs into hollow, bundle-like columns, which are combined into a tower structure. The sophisticated procedure, reliable binding techniques, and ingenious principles of the Tianche craft enable the construction of timber structures exceeding 110 m in height. The craft principles of Tianche provide the following three advantages:

1. Achieving a "self-locking structure" through "interlocking wedging."
2. Enhancing "compression member stability" based on "frictional force transmission."
3. Increasing the "sectional moment of inertia" by using "hollow cylindrical column."

These advantages enhance the structure's strength and stability, enabling the construction of 100-m-tall timber structures using pre-industrial techniques. This study identified the following limitations of the traditional Tianche structures:

1. Susceptibility of the inverted dragon member's tenon to break.
2. Limitations on the column diameters.

The vulnerability of the tenon to break, however, ensures the safety of the overall structure.

In summary, this study investigated the endangered Tianche timber structures, which may contribute to enhancing research on the Tianche craft and could lay the foundation for future conservation practices related to Tianche heritage.

Supplementary Information

The online version contains supplementary material available at https://doi.org/10.1186/s40494-024-01290-6.

Additional file 1. Additional details for understanding the Tianche craft. Figure S1 Bamboo pipelines and circular cow dung drying on the wall. Photograph circa 1929. Figure S2 (a) Oxen power assists the Tianche in lifting brine, and the oxen often sustain injuries due to the demands of the workload. (b) Workers are very strong because of long-term labor and consuming the meat of injured oxen. Photograph circa 1938. Figure S3 Tianches have uniquely shaped the city's skyline for centuries. Photograph circa 1929. Figure S4 Sources of main building materials for Tianche. Figure S5 Craft of mortise and tenon of Tianche, (a) Half tenons; (b) Straight tenons. Figure S6 Comparison of photographs of the Dongyuan Well Tianche before 1987 with its current state. Figure S7 In 1888, American missionary Virgil Chittenden Hart sketched a Tianche structure. Figure S8 Traditional tools used in Tianche construction. Table S2 Natural materials and their applications. Table S2 The condition of existing Tianches.

Additional file 2. Video documentation of the construction procedure of a 5-m-tall Tianche, with the author participation.

Acknowledgements
The authors thank Professor Denghu Jing for his valuable suggestions, particularly regarding the craft analysis. They also express gratitude to the traditional craftsmen of Zigong: Congyou Liu (刘从友), Hanzhao Liu (刘汉朝), Yuanyue Liang (梁援越), and Yongming Xiao (肖永明), for their contributions to the construction experiment. As an apprentice to these craftsmen, the author obtained valuable and authentic field data. The authors extend their thanks to the anonymous reviewers for their valuable comments.

Author contributions
ZZ initially identified the topic, conducted fieldwork, analyzed the data, and later authored the paper. HL and ZZ contributed to the conception and design of the work. ZZ and JY collaboratively produced the images and video materials for the paper. All authors read and approved the final manuscript.

Funding
This work was supported by the National Natural Science Foundation of China (NSFC) under Grant Number 52378006.

Availability of data and materials
All data generated and analyzed during this study are included in this published article [and its supplementary information files].

Declarations

Competing interests
The authors report there are no competing interests to declare.

Received: 2 February 2024 Accepted: 21 May 2024
Published: 28 June 2024

References

1. Lin YX, Song LX, Zhong CY, Ma ZY, Chen R, Nie CX. History of Chinese well salt technology. Chengdu: Sichuan Science and Technology Press; 1987. (**in Chinese**).
2. Richardson HK. The salt-wells of Tzuliutsing. Sci Am. 1916;82(Suppl):324–5. https://doi.org/10.1038/scientificamerican11181916-324supp.
3. Zhang XJ, Zhou GR. Draft history of well salt in Sichuan during the Ming and Qing dynasties. Chengdu: Sichuan People's Publishing House; 1984. (**in Chinese**).
4. Liu DL, Zhou ZZ. Study on ancient Chinese well salt tools. Jinan: Shandong Science and Technology Press; 1990. (**in Chinese**).
5. Chen MD. Research on the large woodwork of *Yingzao Fashi*. Beijing: Cultural Relics Publishing House; 1981. (**in Chinese**).
6. Zhang SQ. Discussion on *Cai* as benchmark in the design system of large woodwork in *Yingzao Fashi*. Archit Hist. 2016;12(2):73–81 (**in Chinese**).
7. Chen ZY, Zhu EC, Lam F, Pan JL. Structural performance of Dou-Gong brackets of Yingxian Wood Pagoda under vertical load—an experimental study. Eng Struct. 2014;80:274–88. https://doi.org/10.1016/j.engstruct.2014.09.013.
8. King WS, Yen JYR, Yen YNA. Joint characteristics of traditional Chinese wooden frames. Eng Struct. 1996;18(8):635–44. https://doi.org/10.1016/0141-0296(96)00203-9.
9. Huang H, Wu YT, Li Z, Su ZW, Chen ZX. Seismic behavior of Chuan-Dou type timber frames. Eng Struct. 2018;167:725–39. https://doi.org/10.1016/j.engstruct.2017.10.072.
10. Zhang LG. Ganlan: a synchronic history of Chinese architecture. J Chongqing Archit Univ (Soc Sci Ed). 2000;1(4):1–3 (**in Chinese**).
11. Liu Y. Building woven arch bridges in southeast China. Constr Hist. 2019;34(2):17–34.
12. Chen S, Yang Y, Shen Z. Impact of new construction activities of Min-Zhe wooden arch bridge in the conservation of its traditional building craftsmanship. Int J Archit Herit. 2023;17(8):1207–20. https://doi.org/10.1080/15583058.2021.2023694.
13. Chen S, Yang Y, Shen Z, Javanmardi A. Reconstruction of Min-Zhe wooden arch bridges and its legitimation as tangible and intangible heritage. Int J Archit Herit. 2022;16(12):1779–96. https://doi.org/10.1080/15583058.2021.1908444.
14. Zigong City Committee for Historical and Cultural Research. Selections of historical and cultural materials of Zigong, vol. 1–5. Zigong: Zigong City Committee for Historical and Cultural Research; 1982. p. 220–30 (**in Chinese**).
15. Su S. Dongpozhilin. Beijing: Zhonghua Book Company; 1981. p. 76–7 (**in Chinese**).
16. Gill WJ. The River of Golden Sand: being the narrative of a journey through China and Eastern Tibet to Burmah. London: J. Murray; 1883.
17. Hart VC. Western China: a journey to the great Buddhist centre of Mount Omei. Boston: Ticknor; 1888.
18. Luo M. The salt well derricks of Sichuan. Well Mine Salt Technol. 1988;16(1):30–4 (**in Chinese**).
19. Ziliujing Salt Factory. Chronicle of Ziliujing Salt Factory. Chengdu: Sichuan People's Publishing House; 1994. p. 4 (**in Chinese**).
20. Li J. Yingzao Fashi, vol. 3. Hangzhou: Zhejiang Photography Publishing House; 2020. p. 196–7 (**in Chinese**).
21. Qiu DH, Hu XK, Chen ZA, Qin F. Materials mechanics. Beijing: Higher Education Press; 2004. (**in Chinese**).
22. Xiao YM. The salt well deep drilling and extraction techniques of Zigong, China. Zigong: Culture, Broadcasting and News Publishing House; 2015. p. 63 (**in Chinese**).
23. The Engineering Tool Box. Friction coefficients for some common materials and materials combinations. https://www.engineeringtoolbox.com/friction-coefficients-d_778.html. Accessed 5 Apr 2024.
24. SWR Steel Wire Rope. 7×7 (6/1)—wire strand core wire rope. https://www.steelwirerope.com/index.html. Accessed 5 Apr 2024.
25. Du SQ. The application of bamboo in well salt production. Res Hist Salt Ind. 1988;2(3):79–80 (**in Chinese**).
26. Rao KMM, Rao KM. Extraction and tensile properties of natural fibers: Vakka, date and bamboo. Compos Struct. 2007;77(3):288–95. https://doi.org/10.1016/j.compstruct.2005.07.023.
27. Liu Y. Woven arch bridge: histories of constructional thoughts. London: Routledge; 2020.
28. Leonardo da Vinci. Codex Atlanticus. https://codex-atlanticus.ambrosiana.it/#/Detail?detail=69. Accessed 20 May 2024.