Sun *et al. Heritage Science* (2024) 12:353
https://doi.org/10.1186/s40494-024-01469-x

Heritage Science

RESEARCH Open Access

How to interpret Jiangnan gardens: a study of the spatial layout of Jiangnan gardens from the perspective of fractal geometry

Ce Sun[1,2], Zhenyu Jiang[3*] and Bingqin Yu[1*]

Abstract

This study contributes to design studies by offering a novel approach to understanding the spatial layout of Jiangnan gardens through the lens of fractal geometry. Analyzing 106 gardens, we found that the ideal fractal dimension range for Jiangnan gardens is 1.148 ~ 1.276, with gardens in the 2500 ~ 7200 m^2 range exhibiting the highest complexity (1.238 ~ 1.276). Additionally, for gardens ranging from 2500 ~ 20,000 m^2, the maximum spacious space area stabilizes, no longer expanding indefinitely with overall area. This suggests a design principle of spatial proportion and balance. By quantifying spatial complexity and the contrast between spacious and profund spaces, the study provides a new method for evaluating garden design and can help students and designers better apply the principles of Jiangnan garden design.

Keywords Jiangnan garden, Spatial layout, Fractal geometry, Fractal dimension, Multifractal spectrum, Spacious space, Profund space

Introduction

Why study Jiangnan gardens?

The history of Chinese gardens can be traced back to the Xia Dynasty in 2070 BC. Over the subsequent nearly 4000 years, Chinese gardens developed along two main lines. One was dominated by imperial gardens in the north, which were typically vast and opulent, serving the emperors. The other was characterized by the literati gardens in the Jiangnan region, with the owners of these gardens typically being members of the literati class. These gardens were generally of smaller scale, yet their interiors were rich in variety and change.

Surprisingly, in the history of Chinese gardens, the Jiangnan gardens constructed by scholar-officials held an unparalleled exalted status compared to imperial gardens. Throughout many periods, Jiangnan gardens were the highest standard for assessing the artistic creation of gardens in society, and they were also frequently imitated by numerous imperial gardens in the north [1]. For instance, the Xiequ Garden in Beijing, built by Emperor Qianlong of the Qing Dynasty, was modelled after the Jichang Garden in Wuxi, Jiangsu. Additionally, within the Yuanmingyuan in Beijing, three gardens were respectively modelled after the Anlan Garden in Haining, the Zhan Garden in Jiangning, and the Quyuan in Yangzhou. The fact that even emperors were so enamoured with Jiangnan gardens attests to their dominant position in Chinese garden history, nearly becoming synonymous with Chinese gardens themselves.

---

*Correspondence:
Zhenyu Jiang
1549443658@qq.com
Bingqin Yu
yubingchin1983@sjtu.edu.cn
[1] School of Design, Shanghai Jiao Tong University, No.800 Dongchuan Road, Minhang District, Shanghai 200240, China
[2] Center for Chinese Classical Garden Research and Design, School of Art and Archaeology, Zhejiang University, No. 148 Tianmushan Road, Xihu District, Hangzhou 310028, Zhejiang Province, China
[3] Gemmological Institute, China University of Geosciences (Wuhan), No. 388 Lumo Road, Hongshan District, Wuhan 430074, Hubei Province, China

Interpreting the two dimensions of Jiangnan gardens: social culture and spatial form

For design research, it is essential to comprehend the local character of a phenomenon by understanding the multiplicity of powers, processes, and practices that constitute it [2]. Jiangnan gardens, as historical heritage, are interpreted through two dimensions: social culture and spatial morphology [3].

People often regard gardens, like other "Things" (such as clothing, calligraphy, and other daily necessities), as products of their specific era. For instance, Craig Clunas [4] studied the cultural significance of Jiangnan gardens within the context of the Ming Dynasty. In his book "Superfluous Things," he utilized Pierre Bourdieu's "La Distinction" theory to argue that Jiangnan garden art was a distinctive class marker for the scholarly elite during the late Ming period [5]. The approach of situating gardens within their social and cultural contexts implies that without understanding the social culture of the time, it is challenging to interpret Jiangnan gardens [6].

However, this is not absolute. For those experiencing the gardens firsthand, the pleasure derived from the spatial arrangement can be felt regardless of their knowledge background. The 18th-century British architect William Chambers noted that the imagery conveyed in Jiangnan gardens is related to subjective bodily experiences, and not greatly influenced by whether the observer possesses the relevant knowledge, "Meadows, woods, shrubs, rivers, and mountains affect both people (those with knowledge and those without) in the same way, and each combination of these elements will excite a similar feeling in both" [7].

Similarly, Peter Jones [8] found that visitors to gardens with diverse spatial arrangements were inclined to explore the paths and were drawn to sit in pavilions for contemplation.

Therefore, it is natural for people to seek connections between subjective experiences and spatial morphology [9]. What is the relationship between these subjective feelings and the spatiality of Jiangnan gardens (or, more specifically, their spatial morphology)? Typological studies of space inform us that spatial morphology has a direct impact on bodily perception [10]. For instance, the spherical atrium space in classical architecture often evokes a sense of solemnity and majesty, which has even evolved into a distinct architectural style [11]. As for medieval gardens, Renaissance gardens, or neoclassical gardens, they all exhibit regularized forms with most prominently featuring axial symmetry, all of which are intended to showcase the grandeur and majesty of royal gardens.

However, typological studies of Jiangnan gardens encounter difficulties. The reason lies in the fact that, on the one hand, the spatial layout of these gardens does not conform to the regular geometric characteristics, being "disorderly" [12], which makes it difficult to extract universal patterns from their geometric forms. On the other hand, the important buildings and artificial constructions within these gardens, whether observed from plan or elevation, show no significant formal differences. Chinese architect Tong Jun [13] and British architect Banister Fletcher [14] in his work "Sir Banister Fletcher's Global History of Architecture" have both expressed similar views: Chinese gardens and architecture, whether temples, tombs, public, or private gardens, only differ in scale and not in form. This is the reason why Fletcher even believed that Chinese architecture is monotonous, having made no progress since ancient times, being merely an industry rather than art. It is evident that traditional Chinese spatial forms exhibit irregularity in form, which renders traditional typological studies unfeasible and has led to misunderstandings of Chinese garden art.

In response to the irregular spatial morphology of Chinese gardens, especially those in Jiangnan, methods from the field of mathematics, such as topology and fractal geometry, have been introduced. The advantage of these approaches lies in their ability to describe the irregular or fragmented shapes of natural features and other complex objects that cannot be analyzed using traditional Euclidean geometry. Topology's strength lies in its ability to abstract irregular forms into regular geometric shapes [15], emphasizing the parallel relationships between spaces [16], as exemplified by Cai's study of the spatial organization within the Humble Administrator's Garden, which attempted to generate garden spaces and paths using computer intelligence [17]. However, this method remains in the exploratory stage.

In contrast to topology, fractal geometry places greater emphasis on describing the interrelationships between the parts and the whole of an object. The most significant feature of fractals is their self-similarity, where, regardless of whether the object is magnified or reduced, the local and the global structures exhibit similar patterns. Thus, fractals are often defined as "shapes composed of parts that are similar or identical to the whole" [18]. In spatial research, fractals manifest as the process of how a larger space is subdivided into multiple smaller spaces, and these smaller spaces can continue to be subdivided, creating a self-similarity and "complexity" in the spatial layout [19]. Fractal Dimension, as an important indicator of fractal geometry, describes this "complexity." It has been widely used in the study of spatial patterns and transportation networks in urban planning [20,21]. Batty and colleagues' simulation analysis suggests that an ideal city form should have a fractal dimension of approximately 1.7 [22]. Lin determined the fractal dimension of

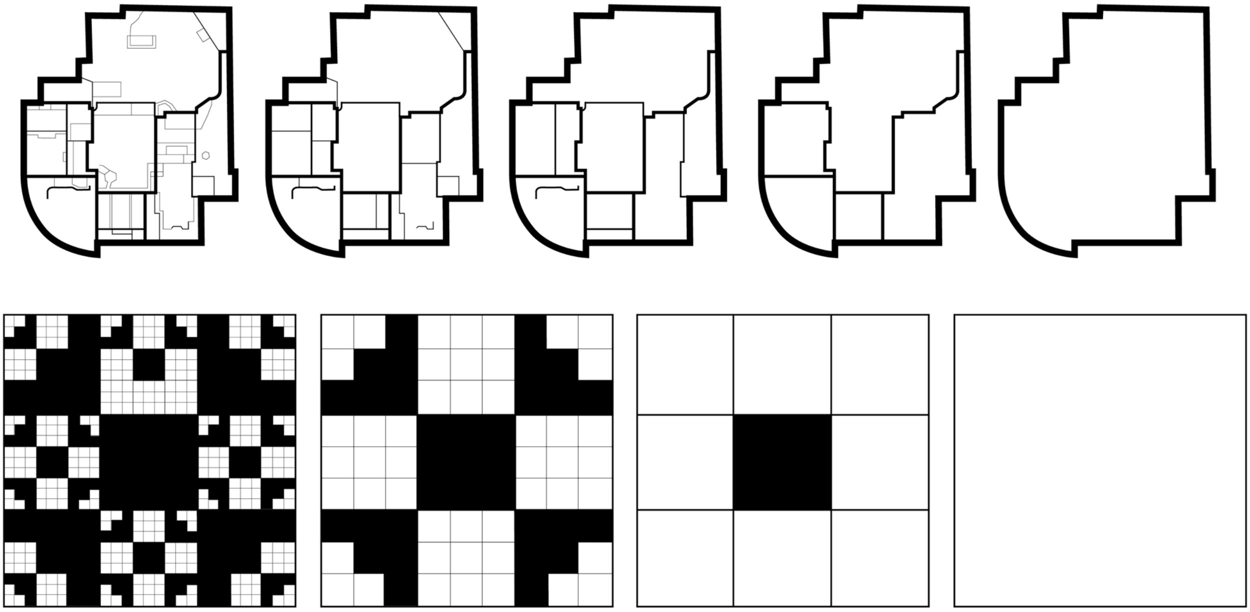


Fig. 1 The spatial layout of Shanghai Yu Garden exhibits self-similarity (provided by Lu Shaoming)

the historic city of Krakow in Poland, which was found to be 1.79 [23]. This indicates that the fractal dimension can serve as an important indicator of spatial complexity and can validate the rationality of the complexity of spatial layouts.

In addition, the multifractal spectrum is another key indicator in fractal geometry. For certain complex structures, they do not adhere to a single fractal law but are often the result of a combination of multifractal characteristics [24]. For complex structures, the multifractal spectrum can describe the characteristics of local structures, allowing for a more precise analysis by examining the differences in characteristics across various local spaces [25].

The possibility and existing problems of fractal geometry interpretation of Jiangnan gardens

In the context of studies on Jiangnan gardens, Chen revealed the relationship between the fractal dimension of buildings in gardens and visual perception [26]. In addition, Lu [27, 28] has noted that the spatial configurations formed by the enclosure of structures such as buildings, corridors, and walls within these gardens exhibit characteristics of self-similarity (Fig. 1). This suggests that the spatial layout of Jiangnan gardens adheres to the principles of fractal geometry, thus rendering it feasible to study Jiangnan gardens from the perspective of spatial morphology.

However, there are still some limitations: (1) The fractal geometry research on the spatial layout of Jiangnan gardens remains qualitative, focusing solely on their self-similarity and not yet delving into the calculation of related indices such as fractal dimension and multifractal spectrum; (2) The sample size is relatively small. Jiangnan gardens are numerous and vary greatly in size. These limitations result in a lack of universality and in-depth quantitative conclusions in the study of Jiangnan garden spatial layouts, rendering it unable to provide insights for the spatial design of Jiangnan gardens.

Therefore, this paper has delineated the spatial layouts of 106 Jiangnan gardens. These spatial layout images are predominantly composed of artificial constructions such as buildings, walls, corridors, and other elements. Other natural elements, such as rocks, water bodies, and plants, have not been included in this study due to the temporary absence of their self-similarity. We have drawn upon the role of fractal dimension in urban planning as a measure of spatial complexity. By categorizing Jiangnan gardens at different scales and calculating the fractal dimensions of these 106 gardens at these scales, we can obtain standard numerical values for the spatial complexity of Jiangnan gardens.

Furthermore, for gardens with more complex spatial layouts, it is necessary to calculate and plot their Multifractal spectrum to investigate the different fractal characteristics of their various components. For Jiangnan gardens, the differences in fractal characteristics of their parts are mainly manifested in the variation of spatial complexity. This difference, when we cannot quantify it, is often referred to as "spacious space" and "profund space," corresponding to the subjective feelings of "spaciousness" and "profundity," respectively. In the design process of Jiangnan gardens, the contrast between spacious space and profund space is a crucial element in creating the artistic essence of Jiangnan gardens. However, such subjective descriptions often fail to convey accurate information in communication, especially in the context of teaching garden design courses. With the aid of the Multifractal spectrum, we can quantify the differences in quantity and area between Spacious space and Profund space in complex gardens, as well as the disparities in complexity between the two spaces. By comparing these numerical values, we can demonstrate the reasonableness of the ratio of Spacious space to profund space in the design.

The calculation of fractal dimension and Multifractal spectrum is of significant importance for the protection, design, and education of Jiangnan gardens. These metrics can also serve as standards to evaluate whether students' garden design works in university education meet these criteria, and to provide corresponding guidance and improvements. This provides a new evaluation method for garden design education, aiding students in better understanding the design principles of Jiangnan gardens and enhancing their design capabilities.

Methods and materials

Materials

We utilised the floor plans of 106 Jiangnan gardens, which were primarily surveyed based on the extant physical gardens. We extracted the spatial layouts formed by the enclosure of buildings and artificial constructions, while other elements such as water, plants, and paths within the gardens were considered as components of the open spaces and were not specifically marked. The resulting drawings were used for the calculation of fractal dimension and the creation of multifractal spectra. The research subjects encompassed gardens in most of the Jiangnan region, including those in Zhejiang, Shanghai, Suzhou, and Yangzhou.

Furthermore, we prepared 6 course design plans for Jiangnan gardens, and through comparing their fractal dimension and Multifractal spectrum values with those of the 106 Jiangnan gardens, we identified the directions for design improvements.

Methods

Fractal dimension and multifractal spectrum

1. Fractal dimension

The fractal dimension can be used to quantify the complexity of geometric structures, their scaling characteristics and self similarity, as well as the effective ratio of surface area to volume [29]. The description of a fractal geometric object is typically completed by its fractal dimension and hierarchy. Unlike two-dimensional or three-dimensional spaces, the dimension of a fractal geometric object must be measured to be determined, thus, the dimension of a garden plan must necessarily fall between 1 and 2.

The most significant characteristic of fractals is their self-similarity. However, in the study of real-world problems in nature, spatial morphology is not as precise as in the mathematical context of fractals, and their self-similarity often focuses more on the similarity of traits. Therefore, such spatial studies can only be described through a method of gradual approximation. Among these, the Box Counting Method is one of the more commonly used methods.

The operational method of the Box Counting Method involves covering the measured shape with squares of edge length r and counting the number of squares $N_{(r)}$, required to completely cover the shape. Then, the edge length of the covering squares is reduced to $r/2$, and the process is repeated to count the number of squares required to cover the shape again. The statistical results are represented by the number of square grids existing at different edge lengths r, and the logarithms of $N_{(r)}$ and $\ln r$ are taken, plotted on a double logarithmic scale to form a statistical curve. The slope of the approximate straight line in the middle of the curve (typically negative due to the slope calculation, hence the opposite number of the slope is used for convenience in description) represents the fractal dimension D. A higher value of D indicates a greater degree of fractality:

$$D = -\log_r N_{(r)} = -\frac{\ln N_{(r)}}{\ln r}$$

2. Multifractal analysis and multifractal spectrum

Multifractal Analysis is a method used to study the distribution of self-similarity in complex structures, with the resulting output being the multifractal spectrum.

Multifractal spectrums describe the interrelationship between the "fractal dimension differences" among various parts of a complex structure and the collection of subsets with the same fractal dimension differences [30]. The fractal dimension differences are represented by

α (Singularity Exponent), and the collection of subsets with the same fractal dimension differences can be correspondingly represented as $f(\alpha)$, indicating the number of subsets with the same level of complexity. The multifractal spectrum is thus expressed as:

$$f(\alpha) \sim \alpha$$

Multifractal spectrums can also be calculated using the Box Counting Method. The principle involves dividing the plane into many boxes of size r (where r<1), with each box defined by its coordinates (i,j), P_{ij} represents the probability distribution within the (i,j) box. If the probability distribution is multifractal, then:

$$P_{ij}(r) \sim r^{\alpha}$$

α known as the Singularity Exponent, is used to describe the local singularity of a fractal object at various scales. It reflects the nature of the probability distribution of the spatial area within the fractal object as it varies with the size of the small boxes r. As α increases, the probability, P of the subsets decreases (since $r < 1$). If the number of boxes on the fractal with the same α is denoted as $N_{\alpha}(r)$, it generally increases as r decreases. If the spatial area distribution is considered to be multifractal, then $N_{\alpha}(r)$ can be written as:

$$N_{\alpha}(r) \sim r^{-f(\alpha)}$$

The multifractal spectrum can be calculated using the commonly employed moment representation method in statistical physics, which is denoted as $f(\alpha)$~α, After defining the q order partition function $\chi_q(r)$, the following expression can be obtained:

$$\chi_q(r) = \sum P_{ij}(r)^q = r^{\tau(q)}$$

$$\tau(q) = lim[\frac{ln\chi_q(r)}{lnr}]$$

where q is a real number, and τ (q) is referred to as the partition function. That is, the τ (q) curve is obtained from the slope of the $ln\chi_q(r)$~lnr curve. The linear range of the $ln\chi_q(r)$~lnr curve is referred to as the scale-invariant range. Only when the system exhibits scale invariance within this range can an accurate multifractal spectrum be determined. The following Legendre transformation can be applied to obtain:

$$\alpha = d[\tau(q)]/d_q$$

$$f(\alpha) = \alpha q - \tau(q)$$

Finally, the multifractal spectrum can be plotted based on the obtained $f(\alpha)$~α. From the formulas $P_{ij}(r)$ ~r^{α} and $N_{\alpha}(r) \sim r^{-f(\alpha)}$, it is evident that:

$$\alpha = lnP/lnr$$

$$F(\alpha) = -lnN/lnr$$

When the value of r is fixed, the multifractal spectrum $f(\alpha)$~α describes the relationship between the distribution probability P and the number of boxes N_{α} with the same α.

We will further illustrate this with a square-shaped multifractal example, which can be considered as the spatial layout of a two-dimensional plane of Jiangnan gardens (Fig. 2). Initially, in Fig. 2 (0) where $r = 1$, a line of three equal parts is found on both the horizontal and vertical axes, dividing the square into four blocks. The masses (or areas) of these four blocks are 4/9, 2/9, 2/9, and 1/9, respectively. Based on this, the four blocks are subdivided in the same manner to obtain Fig. 2 (2). This process can be repeated indefinitely.

From the figure, it can be observed that as the levels deepen, the intervals become increasingly fragmented, and the distribution of mass in these smaller intervals becomes more uneven, corresponding toα. For the block in the bottom left corner of Fig. 2 (1), its distribution probability $P = 4/9$, while the P values for the blocks in the top left and bottom right are the same, both being 2/9, so $N = 2$. From this, the numerical values of α and $f(\alpha)$ can be calculated. The corresponding

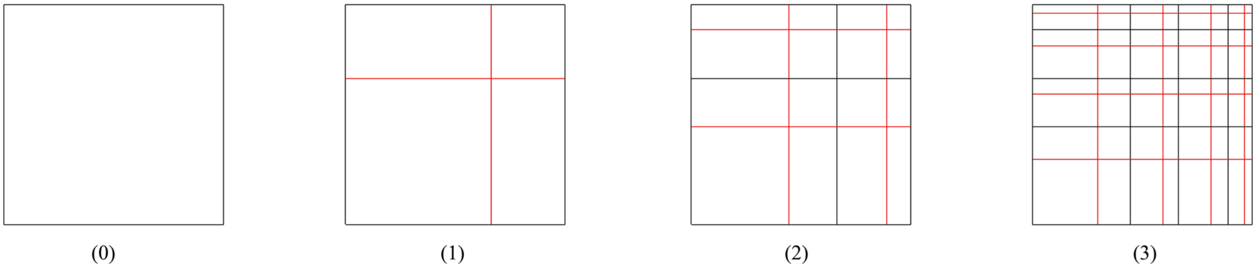


Fig. 2 Square shapes are obtained at different stages based on the same multifractal law

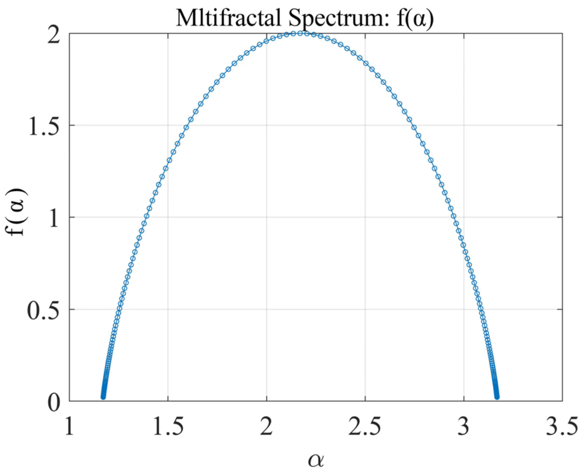


Fig. 3 Multifractal spectra of matrices [2/9, 1/9; 4/9, 2/9]

matrix for the above rules is [2/9, 1/9; 4/9, 2/9], and plotting its multifractal spectrum reveals a symmetric inverted U-shaped curve (Fig. 3).

In an ideal state, the multifractal spectrum is a curve that is symmetrical about the vertical axis. However, in a natural state, multifractals are not homogeneous, which means that some values in $f(\alpha)\sim\alpha$ will be missing, leading to an asymmetrical inverted hook-shaped curve [31]. As a result, in practical applications, $\Delta\alpha$ and $\Delta f(\alpha)$ acquire significance. Since the Box Counting Method uses equal box sizes r for each fractal, and $r < 1$, it follows from $\alpha = lnP/lnr$ that P increases, α decreases. Within the same system, $\Delta\alpha = \alpha_{max} - \alpha_{min} = lnP_{min}/lnr - lnP_{max}/lnr = \ln(P_{min}/P_{max})/lnr$, quantitatively characterizing the difference in the ratio of spatial area (between the most spacious and the most profund spaces). Correspondingly, $\Delta f(\alpha) = f(\alpha_{min}) - f(\alpha_{max}) = -\frac{lnN_{\alpha min} - lnN_{\alpha max}}{lnr} = -\ln(N_{Pmax}/N_{Pmin})/lnr$, which can characterize the proportion of the most spacious and the most profund spaces in terms of quantity.

Operation steps

1. Classification of Jiangnan gardens

Different scales of Jiangnan gardens are not related by uniform scaling on the plane, indicating that the spatial structures of Jiangnan gardens vary at different scales. Therefore, it is first necessary to categorize the 106 research samples based on similar spatial structures but different scales. The study found that they can be divided into the following five categories (Table 1):

We transformed the surveyed plans of gardens into spatial layout plans suitable for analysis, corresponding to five types, and listed one representative example for each type (Fig. 4). It can be intuitively observed that the spatial layout of Jiangnan gardens changes with scale, and the complexity of the spaces also changes accordingly. This complexity requires quantification through the calculation of fractal dimensions.

2. Calculation of fractal dimension

The calculation of fractal dimension can be carried out using ImageJ software. The imported images will be subject to binarization processing, with the box size parameter set to the values of 2, 3, 4, 6, 8, 12, 16, 32, 64, 128. Then the fractal dimension of the image can be calculated in batches. By plotting the data according to the groupings, the "Optimal fractal dimension range" corresponding to Jiangnan gardens at different scales can be obtained.

3. Multifractal analysis & multifractal spectrum

Multifractal Analysis is aimed at research subjects with complex structures, allowing for the exploration of the differences in characteristics among different local spaces. For the five groups of research subjects, the spatial structure of Group 1 is too simple, and there is not much variability between the local spaces in Group 2. Therefore, the fractal dimension can relatively

Table 1 Classification table of Jiangnan gardens

Group	Features	Area (m^2)	Representative
1	A single attached courtyard of a building	(0, 500)	Courtyard 14 of Diguan
2	A community composed of multiple individual buildings, surrounded by multiple courtyards	[500, 2500)	Gaoyi Garden of Tianping Mountain
3	Mainly consisting of orderly and regular architectural courtyards	[2500, 7200)	Wangshi Garden
4	The orderliness of the buildings diminishes, and the garden's spatial layout assumes a free and irregular form	[7200, 20,000)	Central Humble Administrator's Garden
5	Based on natural landscapes, the arrangement of buildings is casually dispersed	[20000, +∞)	Lingyin Temple

(a) (b) (c) (d) (e)

Fig. 4 **a** Group1-Courtyard 14 of Diguan (地官14号庭院) **b** Group2-Gaoyi Garden of Tianping Mountain (苏州天平山高义园) **c** Group3-Wangshi Garden (网师园) **d** Group4-Central Humble Administrator's Garden (拙政园中部) **e** Group5-Lingyin Temple (灵隐寺)

accurately describe the spatial layout characteristics of Group 1 and Group 2, and there is no need to rely on Multifractal Analysis for more in-depth research.

We programmed in MATLAB software to perform multifractal analysis on the binary-processed images, obtaining multiple sets of α and $f(\alpha)$ values. We then plotted the Multifractal spectrum and calculated the values of $\Delta\alpha$ and $\Delta f(\alpha)$.

4. Guidance and improvement for Jiangnan garden design

Steps (2) and (3) yielded the Optimal fractal dimension range and Multifractal spectrum, which provide us with a standard for identifying Jiangnan garden design. We selected six garden plans from the "Jiangnan Garden Design" course at MLA for validation, redrew their spatial layouts, and numbered them as Assignment 1 ~ 6 (Fig. 5). Among them, Assignment 1 ~ 3 correspond to the standard of Group 3, and Assignment 4 ~ 6 correspond to the standard of Group 4.

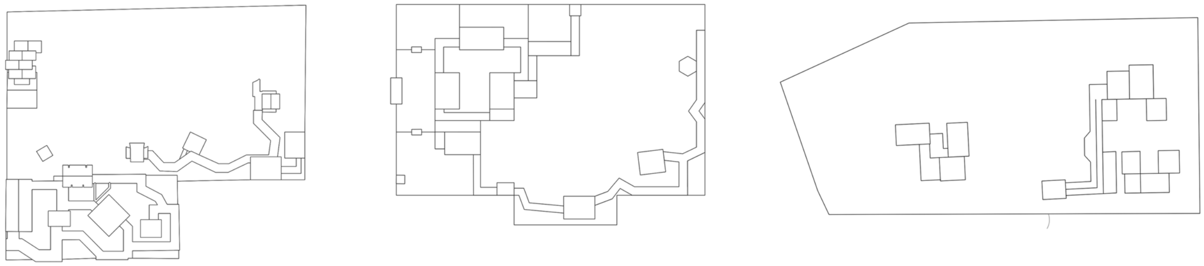
We separately calculated the fractal dimensions and multifractal spectra for the six garden plans, compared them with the "Optimal fractal dimension range" and multifractal spectra at corresponding scales, studied the differences between the two, and proposed corresponding improvement measures.

Results

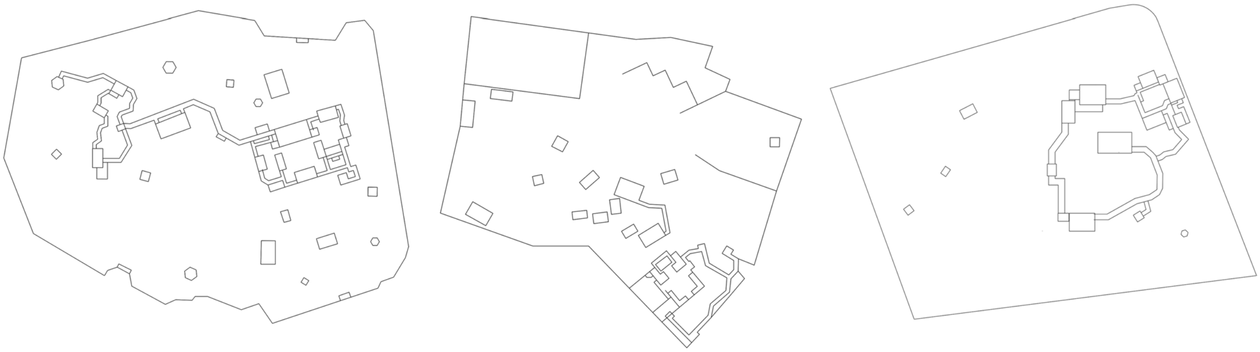
The fractal dimension range of Jiangnan gardens

Measurements of the fractal dimensions of the five groups of Jiangnan gardens (Fig. 6) reveal that the fractal dimension values of Jiangnan garden spatial layouts are distributed within the range of 1.123 ~ 1.329. Additionally, for Jiangnan gardens at different scales, there is a significant difference in fractal dimension values numerically. The Box Plot is drawn based on the quartiles of each group, with its indicators comprising the lower limit, the first quartile (Q1), the median (Q2), the third quartile (Q3), and the upper limit. We can refer to the interval between Q1 and Q3 as the "Optimal fractal dimension range". The larger this interval, the more dispersed the data.

For Group 1, its fractal dimension distribution interval (1.123 ~ 1.205), Optimal fractal dimension range (1.14875 ~ 1.19625), and median (1.18449) are all the lowest. This indicates that the spatial layout of Jiangnan



Assignment 1 Assignment 2 Assignment 3



Assignment 4 Assignment 5 Assignment 6

Fig. 5 Six garden floor plans selected from the "Jiangnan Garden Design" course, Assignment 1–6

gardens with an area less than 500 m^2 is relatively simple. In fact, this can be inferred from the classification of gardens. Since Group 1 consists of independent courtyards around independent buildings, compared to other scales of gardens, their spatial layout is necessarily the simplest. As the garden area increases, we can also observe that the fractal dimensions of each group from Group 1 ~ 3 continue to increase. However, after Group 3, the fractal dimensions of Group 4 and Group 5 decrease again. Therefore, for Jiangnan gardens with an area of 2500 ~ 7200 m^2 (Group 3), their overall spatial structure is the most complex, with the maximum fractal dimension in Group 3 reaching 1.329, the Optimal fractal dimension range being 1.238 ~ 1.276, and the interval being smaller, with a more concentrated overall distribution.

For Group 5, its values are relatively scattered, and the fractal dimension interval is large, with its spatial layout characteristics not being particularly significant. This indicates that Jiangnan gardens with a natural-based foundation and an area greater than 20,000 m^2 have a more free and random spatial layout, which requires appropriate design based on the specific terrain and surrounding environment. Due to the larger randomness in its values, no multifractal analysis was conducted on the Jiangnan gardens in Group 5.

Quantitative results of spacious space and profund space in Jiangnan gardens

Through multifractal analysis and the plotting of Multifractal spectra for Groups 3 and 4, it was found that the Multifractal spectra of Jiangnan gardens both exhibited an asymmetrical inverted hook shape, with the left side shorter and the right side longer (Figs. 7, 8). Numerically, this is due to $\Delta f(\alpha) < 0$ (Table 2). As deduced from the previous formulas, $\Delta f(\alpha) = -\ln(N_{Pmax}/N_{Pmin})/lnr$, If $\Delta f(\alpha) < 0$, then $N_{Pmax} < N_{Pmin}$. This indicates that in Jiangnan gardens, the number of profound spaces is generally greater than that of spacious spaces. Comparisons revealed that for both the first quartile, median, or third quartile, the $\Delta f(\alpha)$ values for Group 3 were lower than those for Group 4 (Fig. 9). This suggests that compared to Group 4, in the Jiangnan gardens of Group 3, the difference in the number of profound spaces and spacious spaces is greater.

Correspondingly, $\Delta\alpha$ describes the ratio difference of spacious space and profound space in terms of area in Jiangnan gardens. The larger the $\Delta\alpha$, the smaller the area difference between spacious space and profound space. From the $\Delta\alpha$ distribution of Groups 3 and 4 (Fig. 10), the median and third quartile are relatively close, with a difference of about 0.1. When considering the first quartile,

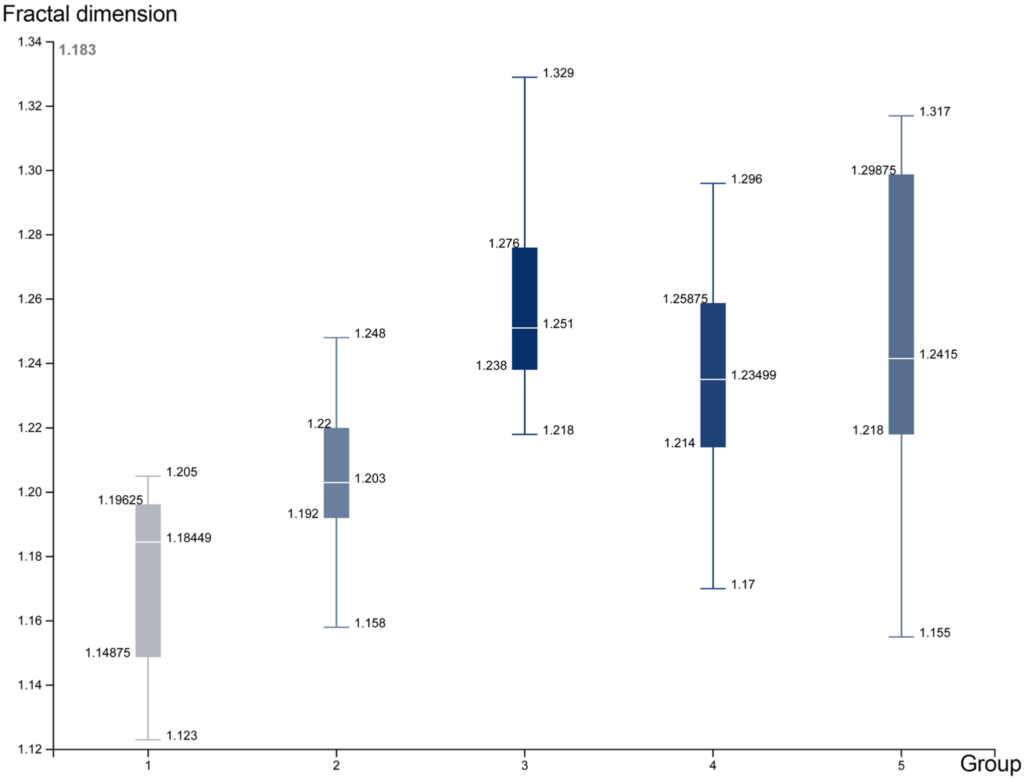


Fig. 6 The fractal dimension distribution of 5 groups

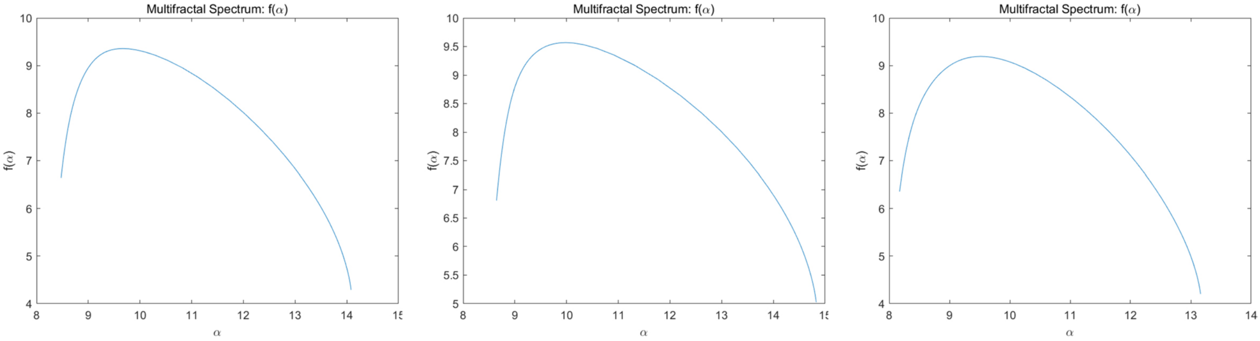


Fig. 7 Three representative cases in Group 3: Couple Garden (耦园), Zhi Garden (芝园: 胡雪岩故居), Zuibaichi (醉白池)

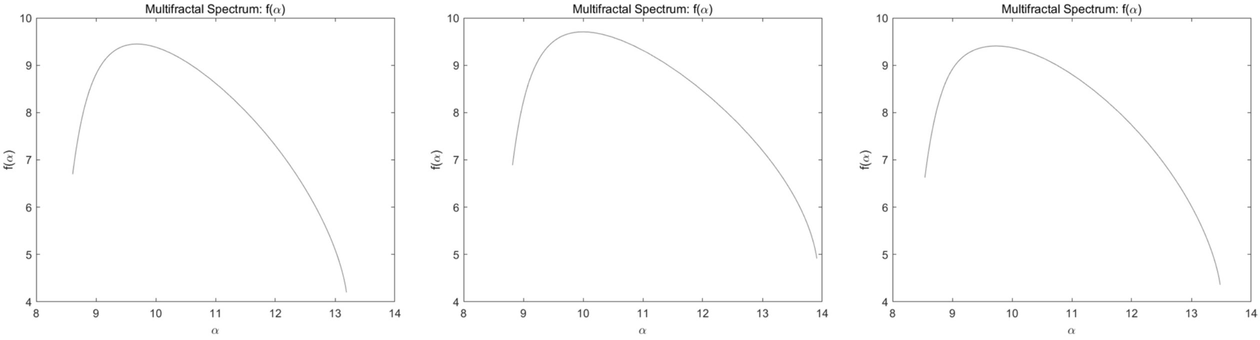


Fig. 8 Three representative cases in Group 4: He Garden (何园), Lion Grove Garden (狮子林), Yu Garden (豫园)

Table 2 Numerical values of multifractal spectra for Group 3 and Group 4

Garden	Group	$\Delta\alpha$	$\Delta f(\alpha)$
Qixing Cave (七星岩)	3	4.9297	− 1.2271
Tingfeng Garden (听枫园)	3	4.3823	− 2.9903
Sui Garden (南京随园)	3	4.6078	− 2.0971
Dongting Dongshan Shude Tang (吴县洞庭东山树德堂东园西宅)	3	5.3134	− 1.0052
Courtyard of Yanyu Building, Jiaxing (嘉兴烟雨楼)	3	5.2306	− 0.9612
Yi Garden (宜园)	3	4.8552	− 1.6372
Courtyard of Shantang Carved Flower Building (山塘雕花楼)	3	5.6502	− 1.1069
Yi Garden, Changzhou (常州意园)	3	4.4805	− 1.7654
Jin Garden, Changzhou (常州近园)	3	4.4501	− 1.9487
Yan Garden, Changshu (常熟燕园)	3	5.1562	− 1.9179
Ying Garden (影园)	3	5.5384	− 0.8298
Yi Garden (怡园)	3	5.0119	− 2.0299
Huiyin Garden (惠荫园)	3	4.6203	− 2.2
Bamboo Garden, Yangzhou (扬州个园)	3	4.9853	− 2.7858
He Garden, Yangzhou (扬州何园)	3	5.1627	− 2.0468
Yongcui Villa (拥翠山庄)	3	5.1176	− 1.1663
Chai Garden (柴园)	3	4.4438	− 2.294
Wang’s Garden (汪氏小苑)	3	3.8274	− 3.1043
Hongdong Villa (红栋山庄)	3	3.5956	− 2.2175
Weaving Department West Garden (织造署西园)	3	3.3858	− 2.8313
Wangshi Garden (网师园)	3	5.218	− 2.1982
Xian Garden (羡园)	3	4.4284	− 3.0709
Couple Garden (耦园)	3	5.6076	− 2.354
Yipu (艺圃)	3	5.0432	− 2.5766
Zhi Garden (芝园: 胡雪岩故居)	3	6.1812	− 1.7829
Ke Garden (苏州可园)	3	4.4719	− 2.3703
Mo’s Manor (莫氏庄园)	3	5.4377	− 3.5134
Xilingyinshe, Hangzhou (西泠印社)	3	4.9065	− 1.7017
Tuisi Garden (退思园)	3	4.6807	− 1.8078
Guo Zhuang (郭庄)	3	5.036	− 1.7476
Zuibaichi (醉白池)	3	4.9881	− 2.1537
Courtyard of Huang Rongyuan Hall (黄荣远堂庭院)	3	5.3217	− 1.0555
Huanglong Cave (黄龙洞)	3	5.8474	− 2.3439
Shuihui Garden (水绘园)	4	5.2574	− 1.8323
He Garden (何园)	4	4.5802	− 2.4998
Xinjiang Academy (信江书院)	4	5.2143	− 0.9395
Lanting (兰亭)	4	5.2735	− 1.0731
Xu Garden, Nanjing (南京煦园)	4	4.487	− 2.1023
Zhan Garden, Nanjing (南京瞻园)	4	5.4828	− 2.3291
Nanxun Xiaolian Village (南浔小莲庄)	4	5.8489	− 1.6996
Yanshan Garden, Taicang (太仓弇山园)	4	4.4643	− 2.6412
Yushan Garden, Shaoxing (寓山园)	4	5.1325	− 1.93
Pingshantang Xi Garden, Yangzhou (扬州平山堂西园)	4	5.7118	− 1.7057
Central Humble Administrator’s Garden (拙政园中部)	4	5.1443	− 1.892
Shen Garden (沈园)	4	4.921	− 1.716
Canglang Pavilion (沧浪亭)	4	5.8934	− 1.4422
Qi Garden, Haiyan (海盐绮园)	4	5.1566	− 1.6713
Fung’s Garden (冯氏花园)	4	4.993	− 2.2538
Qian Garden (潜园)	4	4.6918	− 1.0994

Table 2 (continued)

Garden	Group	$\Delta\alpha$	$\Delta f(\alpha)$
Lion Grove Garden (狮子林)	4	5.0903	– 1.9759
Qiuxiapu (秋霞圃)	4	5.1938	– 2.0147
Mi Garden (绍兴密园)	4	5.1505	– 0.7384
Yu Garden (豫园)	4	4.9398	– 2.2667
Zhenjiang Jin Mountain (镇江金山)	4	5.3438	– 1.5097
Slender West Lake Xiaojinshan, Yangzhou (扬州瘦西湖小金山)	4	5.5313	– 0.9199

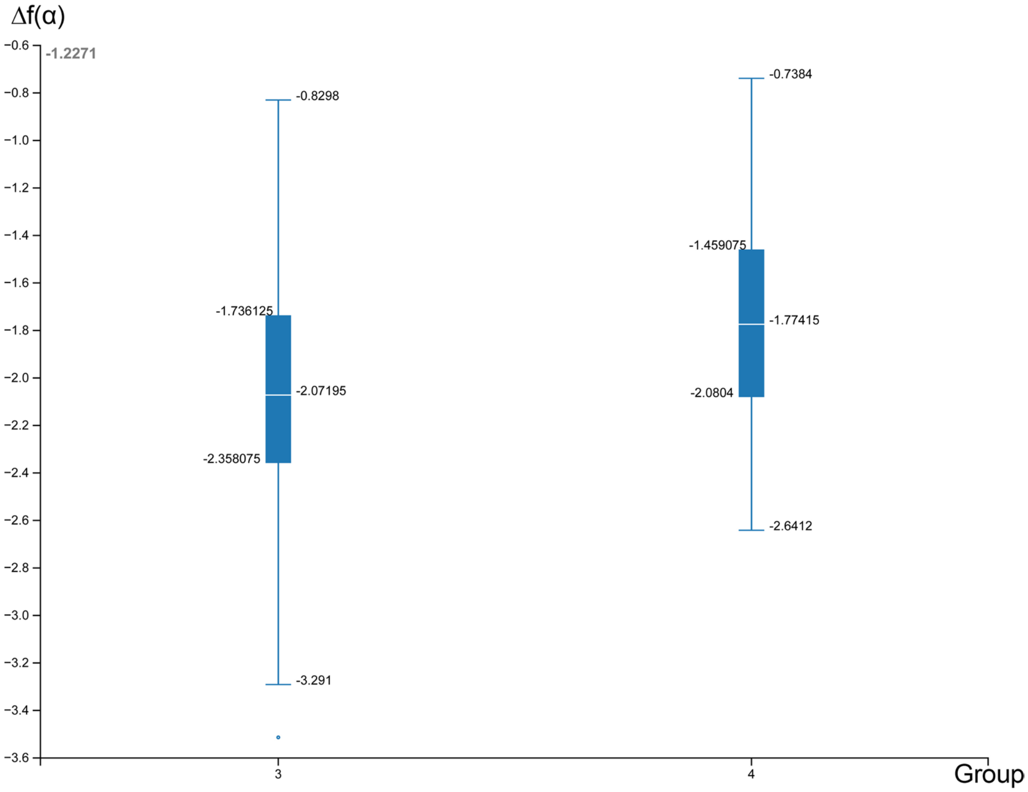


Fig. 9 Distribution of $\Delta f(\alpha)$ values for Group 3 and Group 4

the $\Delta\alpha$ value for Group 3 is generally slightly lower than that for Group 4. This suggests that in the Jiangnan gardens of Group 3, the area difference between spacious space and profound space is relatively large. That is, the contrast between openness and closure in spatial design when the area is between 2500 and 7200 m^2 is more pronounced compared to when the area is between 7200 and 20,000 m^2. From the distribution intervals of $\Delta\alpha$ for the two groups, Group 3 has a wider distribution interval. When $\Delta\alpha$ approaches its maximum value of 6.1812, it indicates that the difference in garden internal space area is small, which from another perspective indicates that the garden's complexity is high, and vice versa (Fig. 11). Group 4's $\Delta\alpha$ distribution interval is narrower, which also suggests that the open and dense spaces in the gardens of Group 4 have a more homogeneous area difference.

It is worth further exploration that, despite a 2 ~ 3 times difference in overall area between Groups 3 and 4, the smallest internal areas of the two gardens are similar. We know that theoretically, a space can be infinitely subdivided, but in reality, the limit of its subdivision is the scale of the human body. If a space cannot be entered by humans, it has no meaning. For example, the dimensions of pavilions in Jiangnan gardens are typically between 2 and 3 m, which is an appropriate scale for the human

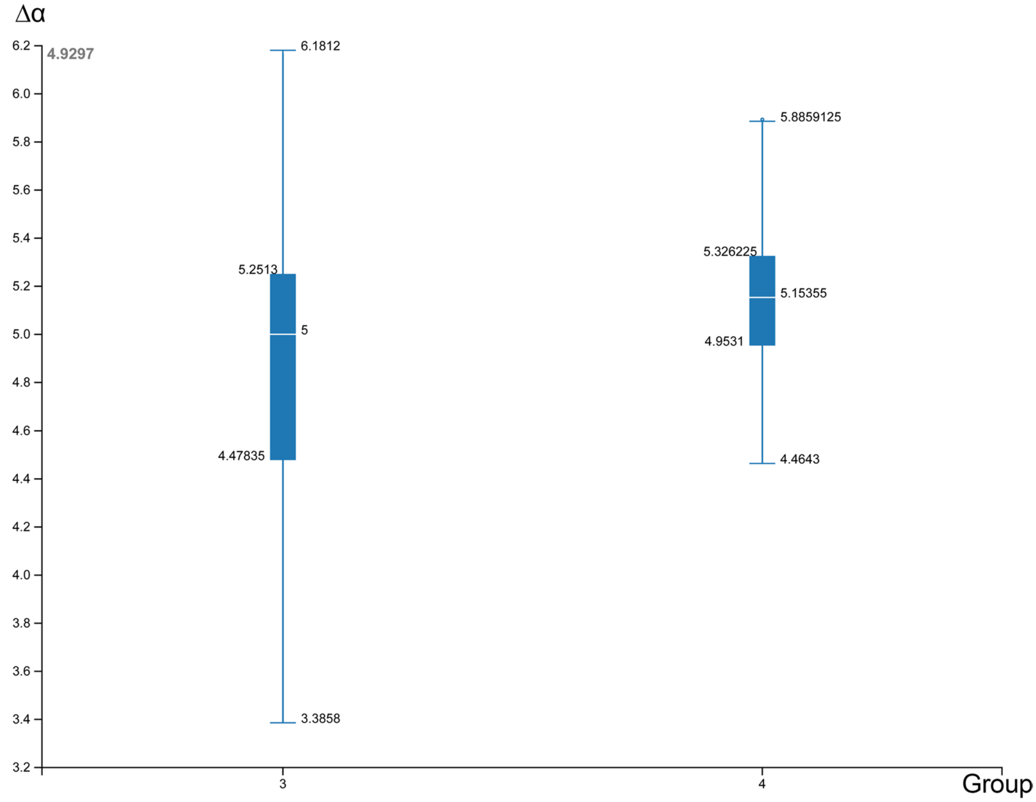


Fig. 10 Distribution of $\Delta\alpha$ values for Group 3 and Group 4

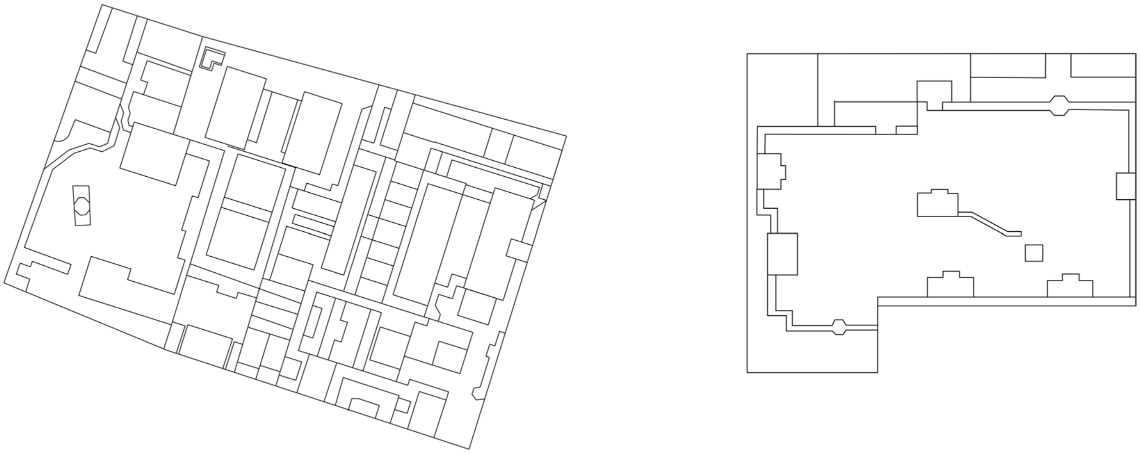


Fig. 11 The two gardens with the largest and smallest $\Delta\alpha$ values in Group 3: Zhi Garden (芝园: 胡雪岩故居) & Weaving Department West Garden (织造署西园)

body. This means that the smallest spaces in Groups 3 and 4 have almost no significant difference in area.

From the above deduction, it can be inferred that $\Delta\alpha = \ln(P_{min}/P_{max})/lnr$, where P_{min} represents the smallest space area, and its value does not have a significant difference. Therefore, only P_{max} (the maximum space area) will affect $\Delta\alpha$. Interestingly, we have found: (1) For Groups 3 and 4, although the garden area of Group 4 is larger, the difference in $\Delta\alpha$ between the two groups is not significant. This indicates that the maximum space area in most of the gardens of Group 4 is the same as that in Group 3, and it does not increase with the overall increase in garden area; (2) the distribution interval of $\Delta\alpha$ within the gardens of Group 3 is larger than that of Group 4, and the difference in $\Delta\alpha$ within Group 3 is greater. This means that the maximum space area within the gardens of Group 4 is relatively homogeneous.

From this, we can infer that when the garden area is between 2500 and 20,000 m^2, the area contrast between spacious space and profound space within the garden does not increase with the increase in garden area. Furthermore, the largest space area within the garden does not increase with the overall increase in garden area. In other words, the overall area of the garden is not a limiting condition for the area of the open space within it. When the overall garden area exceeds a certain value, the difference in the area of the largest space within the garden becomes smaller.

Discussion

Design principles for spatial layout of Jiangnan gardens inferred from the results of multifractal analysis

The results of fractal dimension determination and multifractal analysis reveal some interesting (or counterintuitive) characteristics of Jiangnan gardens. For example, as the overall area of the gardens in Groups 1 ~ 4 increases, the garden with the highest fractal dimension is Group 3, indicating that Group 3 has the most complex spatial layout. Furthermore, from the multifractal spectra of Groups 3 and 4, it can be observed that when the overall area of the garden exceeds a certain value, the largest open space within the garden does not increase with the overall increase in garden area, and the ratio of the area between spacious space and profound space becomes increasingly stable as the overall area of the garden increases.

We attempt to delve deeper into the discussion and speculation on the spatial layout patterns of Jiangnan gardens based on this. The largest open space within the garden does not increase indefinitely as the overall area increases, indicating that in the design of Jiangnan gardens, the spatial layout cannot be simply scaled proportionally to adapt to different-scale sites.

Typically, when faced with a larger design site, the largest open space left after subdividing the site is also larger. Moreover, the larger the design site, the greater the difference in the area of the largest open space between different design schemes. However, the multifractal spectra of Groups 3 and 4 present an opposite phenomenon to this understanding. In the smaller Group 3, the difference in the area of the largest open space in the site is larger and more random. On the other hand, in the larger Group 4, the largest open spaces within different gardens are more homogeneous and have smaller differences. This seems to correspond to some sort of design principle, akin to Le Corbusier's "Modulor" (Fig. 12). If spaces of varying sizes all follow this modulor, the differences in spatial layout would be reduced.

Although we cannot determine which principle the "Modulor" in Jiangnan gardens follows, it can be speculated that the modulor itself already contains spatial variations in openness and closure, and sets requirements for the proportion of the area between large and small spaces. The spatial layout constructed by the modulor differs from the method of spatial subdivision, as it is a design approach that starts small and grows large. That is, it defines the smallest details from the beginning and fills the space with them (Fig. 13).

The method of inserting the modulor also has limitations, as it is restricted by the design site. For instance, when the modulor itself is larger than the site or when the modulor is not suitable for the site (Fig. 14). It can be imagined that the larger difference in the area of the largest open space among different gardens in Group 3 is due to the fact that Group 4 used the method of inserting the modulor, while Group 3 did not. This is because the site area of Group 3 is smaller, which limits the use of the modulor.

Guidance and improvement for Jiangnan garden design

For the 6 garden spatial layout plans from the "Jiangnan Garden Design" course, their fractal dimensions and multifractal spectra were calculated separately, and the results are presented in Table 3.

As discussed in the previous sections, the Optimal fractal dimension range in Groups 3 and 4 are 1.238 ~ 1.276 and 1.214 ~ 1.25875. Within Group 3, the fractal dimensions of both Assignment 1 and 3 are below the minimum threshold of 1.238, whereas the fractal dimension of Assignment 2 is slightly above the maximum value. In Group 4, the fractal dimensions of Assignment 4 ~ 6 all fall short of the optimal fractal dimension threshold. This suggests that the existing spatial layouts for all assignments, except for Assignment 2, are overly simplistic and should be further subdivided to enhance the complexity of the spatial arrangements. Assignment 2, on the other hand, may appropriately reduce the complexity of the spatial layout within its local space.

In terms of the $\Delta\alpha$ values, the optimal $\Delta\alpha$ ranges for Groups 3 and 4 are 4.47835 ~ 5.2513 and 4.9531 ~ 5.326225, respectively. A comparison reveals that Assignment 2 is lower and Assignment 4 ~ 6 are higher. This suggests that the area difference between open space and enclosed space in Assignment 2 is too small and needs to be increased to enhance the spatial

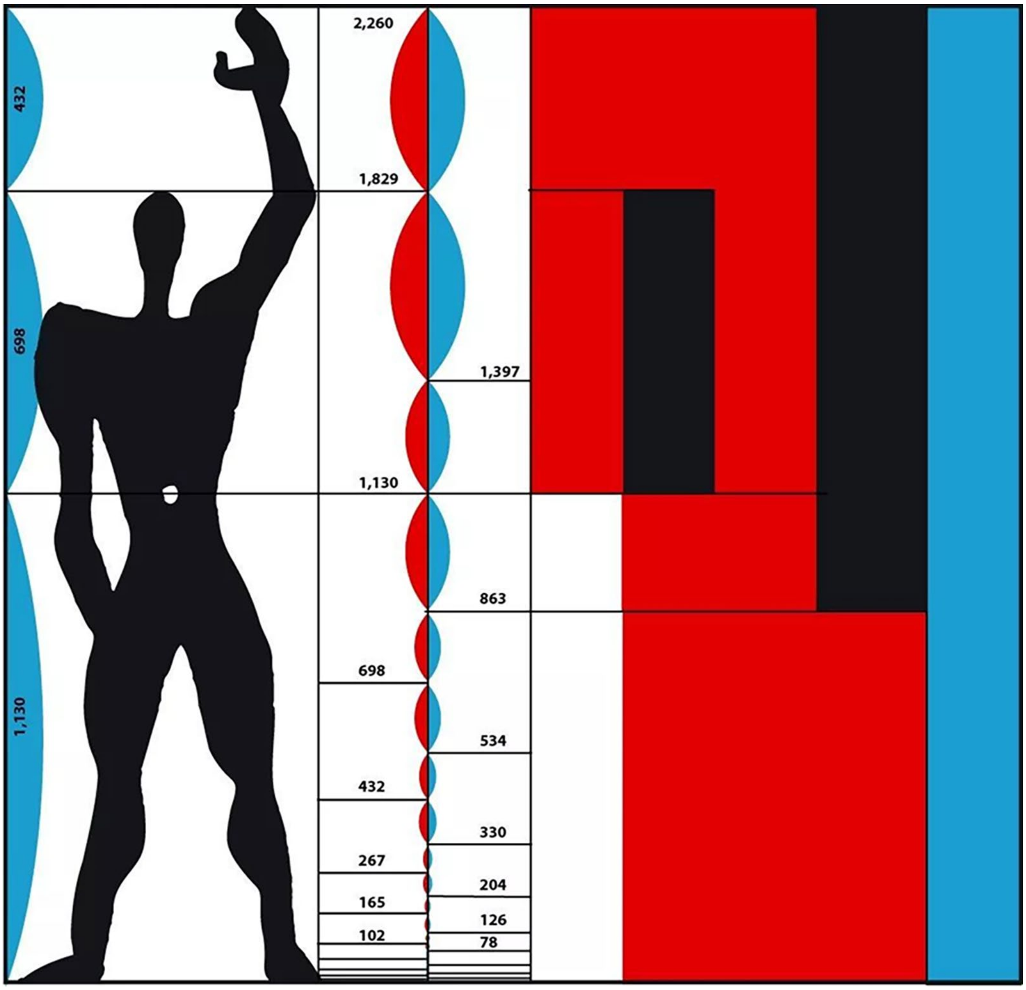


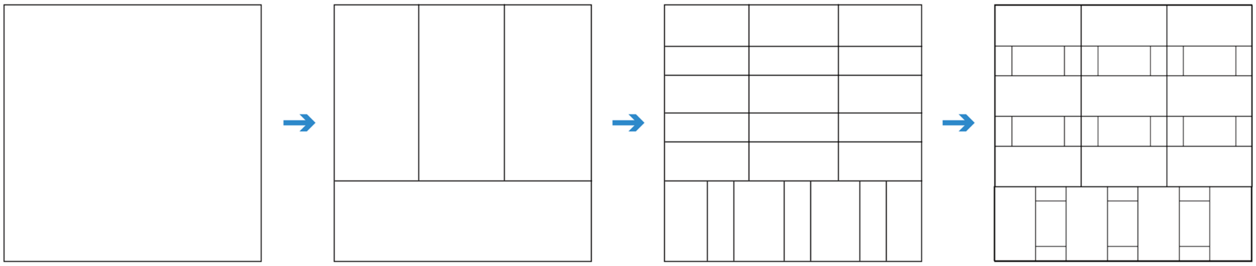
Fig. 12 Modulor, Le Corbusier

difference. Conversely, the spatial difference in Assignments 4 ~ 6 is relatively large.

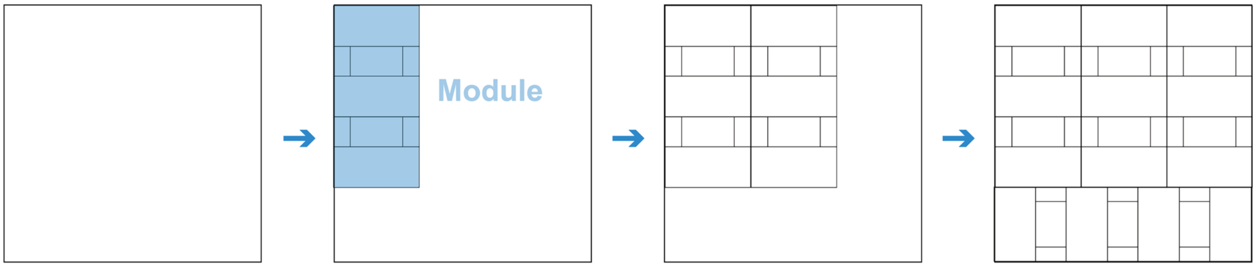
In terms of the $\Delta f(\alpha)$ values, the optimal $\Delta f(\alpha)$ ranges for Groups 3 and 4 are $-2.35807 \sim -1.736125$ and $-2.0804 \sim -1.459075$, respectively. Comparisons show that only Assignment 4 falls within this range, while Assignments 1, 2, 5, and 6 are all larger. Therefore, the number difference between enclosed space and open space should be reduced. Conversely, Assignment 3 has a smaller value, suggesting that the number difference between enclosed space and open space should be increased.

It can be observed that the fractal dimension and multifractal spectrum provide guidance for the design improvement of Jiangnan gardens, yet they do not restrict the specific manifestation of the design. We propose to develop a programme capable of real-time calculation of the fractal dimension and multifractal spectra, which provides intuitive numerical feedback to enable designers to make informed decisions (Fig. 15). As designers sketch the spatial layout, with each stroke added or removed, the three metrics—D, $\Delta\alpha$, and $\Delta f(\alpha)$—are updated accordingly. The programme automatically compares these values to standard benchmarks and suggests directions for improvement, such as "Refine the spatial division to increase spatial complexity" or "Subdivide smaller spaces, or expand the area of larger spaces, to maintain a contrast between small and large spaces," among others.

We invited a student to make spatial layout alterations to Assignment 3, and the programme recorded the changes in the fractal dimensions of the images modified within a 30-min period. Concurrently, we selected six representative nodes to illustrate the planar view of the spatial layout and the associated parameters (Fig. 16). We observed that during this process, the fractal dimension frequently reached the desired range (1.238 ~ 1.276); however, the designer continued to make adjustments. The fractal dimension and multifractal spectrum are, to



The method of spatial segmentation



The method of module insertion

Fig. 13 Two design methods for spatial layout

Module exceeds the site

Module is not suitable for the site

Fig. 14 The modulor method is not suitable for two situations on the site

Table 3 Values of multifractal spectra for assignments 1–6

Garden	Group	Fractal dimension	$\Delta\alpha$	$\Delta f(\alpha)$
Assignment 1	3	1.1811	4.9487	− 1.296
Assignment 2	3	1.1306	4.2515	− 1.2431
Assignment 3	3	1.2022	4.8451	− 2.5486
Assignment 4	4	1.1982	5.4957	− 1.5627
Assignment 5	4	1.2081	5.3791	− 0.9972
Assignment 6	4	1.1877	5.4035	− 1.0019

some extent, merely reference indicators in the spatial layout of gardens, but they are not decisive. Whilst meeting the criteria, the designer must also consider a more multi-dimensional array of influencing factors to achieve an optimal design outcome.

The advantage of this approach lies in the following aspects:

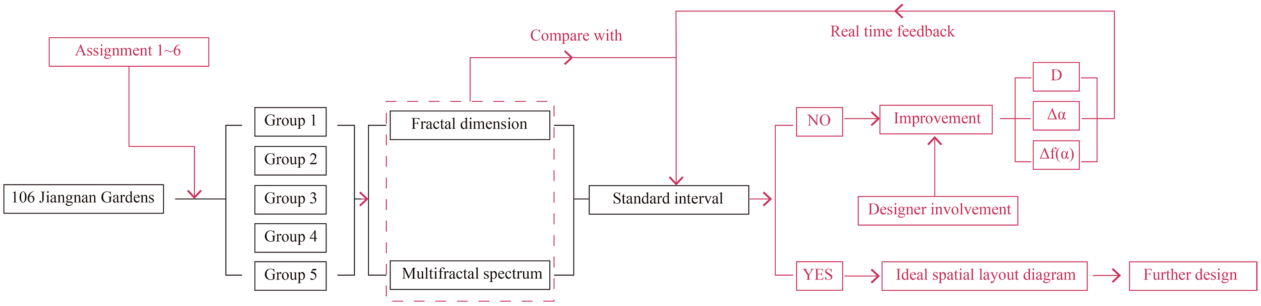


Fig. 15 Real time measurement process diagram of fractal indicators in Jiangnan gardens

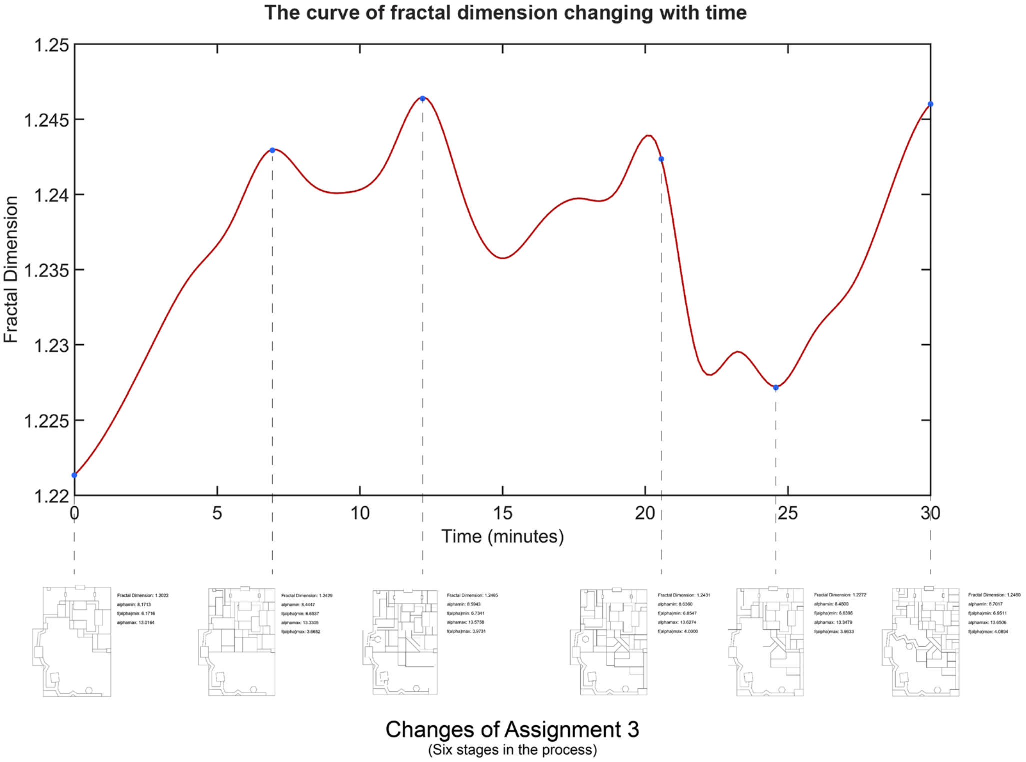


Fig. 16 The curve of fractal dimension changing within 30 min

1) It ensures the designer's subjective initiative. Layouts generated by computers are inevitably constrained by algorithms. Design, however, is a problem-solving process under complex circumstances, where the considerations are multidimensional. We consistently maintain that computers cannot fully comprehend the design process, which requires human subjective intervention.
2) It avoids homogenisation of design. The three metrics are not fixed values but rather a range of intervals. They allow designers to make active modifications within this range. Furthermore, based on the programme's suggestions, once the garden's spatial layout design meets the standards, the designer still needs to refine specific details. For instance, a line on the plan could represent a building or a wall, which should be determined based on the specific function and the designer's aesthetic preferences.

Speculation on artistry of Jiangnan gardens from fractal dimension

The multifractal spectrum represents the difference between the largest open space and the most enclosed space within Jiangnan gardens, essentially characterizing the features of these two extremes. The fractal

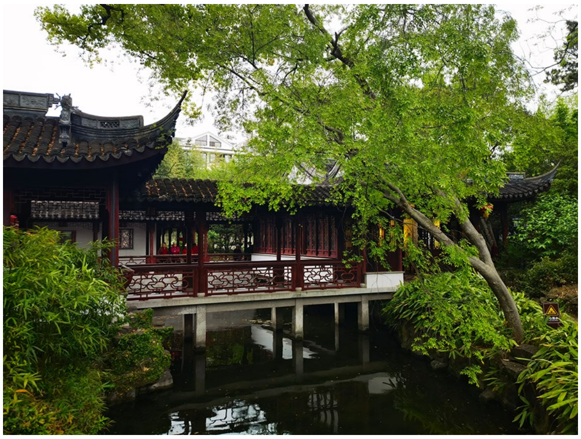


Fig. 17 Multiple small spaces formed by building enclosure, which interpenetrate with each other

dimension, on the other hand, represents the overall condition of the Jiangnan garden. The fractal dimension of Group 3 is greater than that of Group 4, indicating that its spatial layout is more compact. This is also evident from the fact that the $\Delta f(\alpha)$ values for Group 3 are all lower than those for Group 4, suggesting that the number difference between enclosed space and open space is greater in Group 3. This indicates that the spatial subdivision of Group 3's garden is more intricate, tending to create numerous small spaces. One of the important artistic techniques of Jiangnan gardens is "To make a small garden look larger." To achieve this, it is not about the size of individual spaces but about creating multiple spaces. By allowing spaces to transition and permeate into each other (Fig. 17), a sense of openness can be created.

This can be said to be a spatial layout strategy under conditions of limited design sites. The Jiangnan gardens in Group 3 have an area ranging from 2500 ~7200 m^2, which was also a common size for the literati class's homes during the Ming and Qing dynasties. During this period, with the dramatic increase in population density in Chinese society, the building density in these gardens also increased, resulting in a more compact spatial layout. From the comparison between Groups 3 and 4, it can be found that under conditions where the area is not limited, the more suitable fractal dimension for Jiangnan garden space is between 1.214 and 1.25875.

Limitations

The conclusions of the above research, based on the spatial layout samples of 106 Jiangnan gardens, are relatively macroscopic. This is a general case, but it does not necessarily mean that the spatial layout of each Jiangnan garden necessarily applies to the above conclusions. For example, in the measurement of fractal dimension, the lowest value of Group 3 is much lower than the highest value of Group 4. In the gardening treatise "Yuan ye" written by Ji Cheng in the Ming Dynasty, it is mentioned that it is difficult to have universal methods for designing gardens, and differentiated design should be carried out according to different terrains and environments. The conclusions of this article do not rely on the specific spatial morphology of Jiangnan gardens, but focus on the interrelationships between spaces.

Furthermore, Jiangnan gardens are not static over different eras. With the addition, reconstruction, or demolition of buildings, the spatial layout of the gardens will also undergo corresponding changes. For example, Yi Fang and the Simian Pavilion at Zuibai Chi in Shanghai were remodeled by Dong Qichang during the Ming Dynasty, building upon the foundation laid by Zhu Zhichun's "Guyang Garden". Chi Shang Cao Tang was subsequently added by Gu Dashen in the Qing Dynasty, further expanding upon Dong Qichang's contributions. However, in the course of these garden renovations, the details mentioned are but a fraction of the alterations known to us, and a thorough comprehension of the renovation specifics at each location is not fully achievable. Hence, this paper opts to examine the surviving garden plans that offer a comprehensive view of the spatial arrangements.

Conclusions

This paper employs fractal geometry to address the challenge of interpreting the morphology of Jiangnan gardens, representative of Chinese garden art, which is difficult to understand using traditional geometric methods. Based on the characteristics and scale differences of Jiangnan gardens, we categorize 106 research subjects into five groups. The study focuses on analyzing the spatial layout of Jiangnan gardens through two indicators: fractal dimension and multifractal spectrum, aiming to reveal their inherent patterns and design principles. Our findings indicate:

1) The ideal fractal dimension ranges for Jiangnan gardens of different areas are 1.148 ~ 1.196 for 0 ~ 500 m^2, 1.192 ~ 1.220 for 500 ~ 2500 m^2, 1.238 ~ 1.276 for 2500 ~ 7200 m^2, and 1.214 ~ 1.259 for 7200 ~ 20,000 m^2. Fractal dimension characterizes the complexity of the space, with gardens of 2500 ~ 7200 m^2 having the most complex spatial structure. Gardens exceeding 20,000 m^2 do not exhibit distinct fractal characteristics.
2) As the garden area falls within 2500 ~ 20,000 m^2, the largest open space within the garden does not expand indefinitely with the overall area, and the ratio of spacious space to profound space becomes increasingly stable with increasing garden area.

3) The design method for Jiangnan gardens with areas between 2500 ~ 7200 m^2 and 7200 ~ 20,000 m^2 is different. The former employs an approach of cutting large spaces to create small ones from top to bottom, while the latter uses a bottom-up method of spreading the "modulus" to design the site. The "modulus" refers to the proportional and rhythmic changes in the opening and closing of spaces within the garden.
4) Gardens with areas between 2500 and 7200 m^2 exhibit the most compact spatial layouts. This scale allows Jiangnan gardens to create the sensation of "experience the feeling of a large space in a small space" in space. Making a small garden look larger is achieved not by increasing the size of individual spaces but by creating multiple narrow spaces that transition and permeate into each other, thus creating a sense of openness.
5) The fractal dimensions calculated in this paper, along with the $\Delta\alpha$ and $\Delta f(\alpha)$ values from the multifractal spectrum, can serve as a reference standard for the spatial layout of Jiangnan gardens and be applied to course design. Fractal dimension characterizes the overall complexity of the garden space, $\Delta\alpha$ represents the ratio difference in area between spacious and profund spaces, and $\Delta f(\alpha)$ represents the ratio difference in quantity between spacious and profund spaces. Comparing the post-design values of Jiangnan gardens with the standard values can provide direction for improvement.

The study's results are significant for the protection, design, and education of Jiangnan gardens. By quantifying the complexity and density of Jiangnan garden layouts, the study enhances our understanding and evaluation of their artistic value, offering references and guidance for garden design. Additionally, it introduces a new evaluation method for garden design education, aiding students in grasping the design principles of Jiangnan gardens and enhancing their design capabilities.

Supplementary Information

The online version contains supplementary material available at https://doi.org/10.1186/s40494-024-01469-x.

Additional file 1.

Acknowledgements

We would like to express our sincere gratitude to Professor Lu Shaoming for providing suggestions and relevant images for this article, as well as Wang Haozhe, Yang Nan, Peng Zan, and Mu Weihang for providing images for this article.

Author contributions

CS was responsible for most of the work and writing of the manuscript. ZJ were responsible for the data analysis of the manuscript. BY was responsible for the review of the manuscript. All authors read and approved the manuscript.

Funding

The research for this work was supported by the National Social Science Foundation of China project (21BSH124).

Availability of data and materials

Data is provided within the supplementary information files.

Declarations

Ethics approval and consent to participate

Not applicable.

Consent for publication

Not applicable.

Competing interests

The authors declare no competing interests.

Received: 8 August 2024 Accepted: 29 September 2024

References

1. Gu L. Trends in Chinese garden-making: the Qianlong emperor (1711–99) and gardens of Jiangnan. Gard Hist. 2018;46(2):184–95.
2. Vanni I, Crosby A. Place-based methodologies for design research: an ethnographic approach. Des Stud. 2023;85: 101168. https://doi.org/10.1016/j.destud.2023.101168.
3. Lian J, Nijhuis S, Bracken G, Wu X, Wu X, Chen D. Conservation and development of the historic garden in a landscape context: a systematic literature review. Landsc Urban Plan. 2024;246: 105027. https://doi.org/10.1016/j.landurbplan.2024.105027.
4. Clunas C. Fruitful sites: garden culture in Ming dynasty China. Durham: Duke University Press; 1996.
5. Clunas C. Superfluous things: material culture and social status in early modern China. Honolulu: University of Hawaii Press; 2004.
6. Lu A. Lost in translation: modernist interpretation of the Chinese Garden as experiential space and its assumptions. J Archit. 2011;16(4):499–527. https://doi.org/10.1080/13602365.2011.598703.
7. Chambers W, Bartolozzi F. A dissertation on oriental gardening. London: The Royal Academy; 1773.
8. Blundell Jones P, Woudstra J. Social order versus 'natural' disorder in the Chinese garden. Stud Hist Gard Des Landsc. 2014;34(2):151–75. https://doi.org/10.1080/14601176.2013.842066.
9. Liu M, Nijhuis S. Talking about landscape spaces. Towards a spatial-visual landscape design vocabulary. Des J. 2022;25(2):263–81. https://doi.org/10.1080/14606925.2021.2021672.
10. van der Voordt TJM, Vrielink D, van Wegen HBR. Comparative floorplan-analysis in programming and architectural design. Des Stud. 1997;18(1):67–88. https://doi.org/10.1016/S0142-694X(96)00016-6.
11. Tuan Yi-fu. Topophilia: a study of environmental perception, attitudes, and values. Englewood Cliffs, NJ, Prentice-Hall; 1974.
12. Forster AM, Thomson D, Richards K, Pilcher N, Vettese S. Western and Eastern Building Conservation Philosophies: perspectives on permanence and impermanence. Int J Archit Heritage. 2018;13(6):870–85. https://doi.org/10.1080/15583058.2018.1490827.
13. Tong J. Chronicles of Jiangnan Gardens. Beijing: China Architecture & Building Press; 1963.
14. Fletcher B, Cruickshank D. Sir Banister Fletcher's a History of Architecture. 20th ed. Oxford. Boston: Architectural Press; 1996.

15. Zhang J. Pilgrimaging mountains and rivers: the spatial layout of ancient Chinese settlements and their environments. Stud Hist Gard Des Landsc. 2023;43(4):254–78. https://doi.org/10.1080/14601176.2023.2284615.
16. Zhang J. The cultural gene of ancient Chinese space. Beijing: Tsinghua University Press; 2016.
17. Cai C. Intelligent Generation of Architectural layout inheriting spatial features of Chinese Garden Based on Prototype and Multi-agent System—a case study on Lotus Teahouse in Yixing. CAADRIA proceedings; 2019. https://doi.org/10.52842/conf.caadria.2019.1.291.
18. Feder. Fractals. New York: Plenum Press; 1988.
19. Jiang Y. Identify space gene: deal with the phenomenon of all cities with similar urban form. Des J. 2022;25(2):282–90. https://doi.org/10.1080/14606925.2021.2003070.
20. Batty M. The size, scale, and shape of cities. Science. 2008;319:769–71. https://doi.org/10.1126/science.1151419.
21. Peng P, Zhou X, Wu S, et al. An exploration of the self-similarity of traditional settlements: the case of Xiaoliangjiang Village in Jingxing, Hebei, China. Heritage Sci. 2024;12:196. https://doi.org/10.1186/s40494-024-01311-4.
22. Batty M, Longley P. Fractal cities: a geometry of form and function. London: Academic Press; 1994.
23. Lin Q, He Y. Historical space analysis and rehabilitation of Krakow from the perspective of fractal theory. Urban Plan Int. 2020;35(1):71–8.
24. Stanley H, Meakin P. Multifractal phenomena in physics and chemistry. Nature. 1988;335:405. https://doi.org/10.1038/335405a0.
25. Murcio R, Masucci AP, Arcaute E, Batty M. Multifractal to monofractal evolution of the London street network. Phys Rev E. 2015;926:062130. https://doi.org/10.1103/PhysRevE.92.062130.
26. Chen Y, Gu Y, Liu Y, et al. Unveiling the dynamics of "scenes changing as steps move" in a Chinese classical garden: a case study of Jingxinzhai Garden. Heritage Sci. 2024;12:131. https://doi.org/10.1186/s40494-024-01246-w.
27. Lu S. Hidden Orders in Chinese Gardens: irregular fractal structure and its generative rules -the case of Yuyuan in Shanghai, Environment and Planning B. Plan Des. 2010;37(6):1076–94. https://doi.org/10.1068/b3604.
28. Lu S. On spatial complexities of traditional gardens in Jiangnan with the view of fractal geometry and narratives: a case Study of Zuibai Chi. Urban Dev Stud. 2013;20(6):160–5.
29. Datseris G, Kottlarz I, Braun AP, Parlitz U. Estimating fractal dimensions: a comparative review and open source implementations. Chaos. 2023;33(10): 102101. https://doi.org/10.1063/5.0160394.
30. Lopes R, Betrouni N. Fractal and multifractal analysis: a review. Med Image Anal. 2009;13(4):634–49. https://doi.org/10.1016/j.physa.2017.01.041.
31. Hadrien S, Roberto M, Elsa A. Multifractal methodology. Physica A. 2017;473:467–87. https://doi.org/10.1016/j.physa.2017.01.041.

Publisher's Note